Editorial

Learning and Teaching Mathematics: A Cognitive Perspective

Gillian Boulton-Lewis and Lyn English

We are delighted to have had the privilege of being editors for this special issue which has a focus on cognitive perspectives on teaching and learning mathematics. It has taken a long time to come to fruition. It was about 4 years ago that I (Gillian) suggested to the then editor, Nerida Ellerton, that I would be happy to edit an issue with such a focus. In late 1997, I was asked if I would be prepared to edit the special edition for 1998. I agreed enthusiastically, because I believe that it is very important to focus on student cognition and the learning aspects of any teaching situation and that ongoing research and subsequent publications are needed in mathematics education journals in this area. There are many researchers in MERGA who undertake research in cognitive aspects of mathematics education, and the special issue would give some of them an opportunity to present significant aspects of their work.

I asked Lyn if she would be co-editor with me because of her undoubted expertise in the area. Lyn has written summary chapters of work in cognition for MERGA publications which have highlighted the diverse interests of Australian researchers in this field. This special issue is an opportunity to explore some of these interests in greater depth.

More specifically, the present issue indicates the continued appeal of the SOLO (Structure of Observed Learning Outcomes) model for analysing cognitive development in mathematics. The SOLO model was originally advanced by Biggs and Collis (1982) and has since been refined in their later works (e.g., Collis & Biggs, 1991). The use of the SOLO model in research in mathematics is strong in Australia, where it was developed, and this is reflected in the articles in this issue. In this issue, Watson and Mulligan, Watson and Moritz, and Chick apply the model to mathematical learning among young children, older students, and adults, respectively.

Watson and Mulligan present an analysis of young students' development of multiplication and division concepts based on the SOLO model. Their research has identified an increasingly complex range of counting, additive, and multiplicative strategies based on an equal-grouping structure, demonstrating conceptual growth through the ikonic and concrete symbolic modes. The authors present a SOLO developmental model for multiplication and division, described in terms of developing structure and associated counting and calculation strategies.

Watson and Moritz apply the SOLO model to analysing the understanding of chance measurement displayed by students in third, sixth, and ninth grades in Tasmanian schools. Of particular interest in their paper are the documented changes in the observed outcomes in relation to the SOLO model. The detail provided by the model may help teachers determine where their students are currently performing in their understanding of chance, and may also provide information on the students' potential for improvement.

At the other end of the spectrum, Helen Chick applies the SOLO model to a cognitive analysis of a mathematician's output. As she notes in her paper, studies of mathematical cognition at the highest level of formal functioning are scant in the literature. Chick begins to fill this void by considering what characterises learning at this advanced level and how the cognitive complexity of such learning might be addressed in terms of the SOLO model.

The remaining papers in this special issue focus on algebraic learning and geometric problem solving. Students' learning of algebra has been a popular topic of research by Australian mathematics educators. MacGregor and Stacey examine algebraic problem solving by 14-16 year olds, while Pillay, Wilss, and Boulton-Lewis address issues pertaining to students' pre-algebraic understanding. Chinnappan discusses high school students' geometry schemas.

MacGregor and Stacey are concerned with cognitive models underlying algebraic and non-algebraic solutions to unequal partition problems. They found, with a sample of 14-16 year olds, that for a certain class of problems different presentations promoted the construction of different cognitive models. Their data provide support for the hypothesis of Nathan, Kintsch, and Young (1992) that there are three components in the solution of word problems: a propositional text base, a cognitive model, and a formal model of the mathematical relationships. However, they propose that for certain problems there are two valid cognitive models and only one of these is linked to an algebraic representation. They suggest that if the alternative model is chosen, it presents a powerful obstacle to the use of algebraic methods. If teachers are aware of this problem, they can probably help students to overcome it.

Pillay, Wilss, and Boulton-Lewis focus on pre-algebraic understanding and the transition from arithmetic to algebra. The paper examines arithmetic and algebra from a cognitive perspective in order to determine what constitutes a pre-algebraic level of understanding. On the basis of the results of a longitudinal study in grades 7, 8, and 9, a model is proposed for the transition from arithmetic to pre-algebra to algebra, and student results are discussed in the light of this model. A case is made for the need to specifically teach aspects of pre-algebra.

Chinnappan examines the nature of prior mathematical knowledge that would facilitate the construction of useful problem representations in the domain of geometry. This prior knowledge is analysed for organisation in terms of schemas. Problem-solving schemas of high- and low-achieving students were compared and it was found that high achievers accessed more problem-relevant schemas than low achievers and activated a greater proportion of them. It is suggested that high achievers build qualitatively more sophisticated schemas than low achievers and that this helps them construct representations that are more conducive to understanding the structure of geometry problems. Perhaps low achievers could be helped to use better schemas.

All the papers in this issue focus in detail on students' cognitive processes and understanding of selected mathematical concepts. Whilst we know that it is very difficult for teachers to understand their students in such detail, we believe that the research described here should be of great benefit in informing mathematics curriculum design and classroom decision making.

The Process

In October 1997 we sent out a call for articles through the MERGA network and internationally. We had a large number of enquiries and some brief summaries for consideration. Finally we received eight papers. These were each sent out to three reviewers with requisite expertise in the area of interest. They were then returned to authors with our and the reviewers' comments. As a result, six papers were accepted for this issue.

We owe a large debt of thanks to Lynn Wilss, who is a senior research assistant in the School of Learning and Development at Queensland University of Technology. She scrutinised the papers for APA style requirements, checked the references, and then formatted the papers according to *MERJ* guidelines. (Final copy editing was done by the *MERJ* Editor.) Without her efforts, you would have been waiting much longer for this edition to appear.

References

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