

Editorial

The Transition from School to University and Beyond

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The period of transition from school to university is coming under increased scrutiny in the light of growing concerns about decreasing numbers of students opting to study mathematics at university and beyond, and their apparently decreasing levels of competence (Smith, 2004). While the increasing numbers and diversity of those attending higher education institutions may be a factor, the evidence is that those who do opt to study mathematics often find the going difficult, reflected in low first year pass rates at some institutions. This was recorded in the joint report *Tackling the Mathematics Problem* prepared by the UK's Institute of Mathematics and its Applications, the London Mathematical Society and the Royal Statistical Society (1995), which described a serious lack of essential technical facility, a decline in analytical powers and a lack of appreciation of the place of proof in mathematics on the part of undergraduates. While this was written some years ago, in many countries there is still a concern that what is sometimes referred to as a 'gap' (not just temporal) between school and tertiary mathematics, may be increasing. More recently we have seen *Before It's Too Late—A Report to the Nation from the United States National Commission on Mathematics and Science Teaching for the 21st Century*, and, in 2004, *Making Mathematics Count*, a report of Professor Adrian Smith's Inquiry into Post-14 Mathematics Education in the UK. Among other observations the 2000 US report commented "We are failing to capture the interest of our youth for scientific and mathematical ideas. We are not instructing them to the level of competence they will need to live their lives and work at their jobs productively." (pp. 4, 5). Reflecting the current opinion on the importance of transition in mathematics, the recent IMU/ICMI Pipeline Project has been set up to study transition points: a) From school to the undergraduate programme; b) From the undergraduate programme to teacher education (and to teaching); c) From the undergraduate programme to higher degrees (e.g., Masters and PhD) in mathematics; and d) From higher degrees to the workforce.

The collection of papers in this special edition of MERJ addresses precisely the issues of the existence and nature of any difficulties in the transition from school to university study of mathematics, and beyond. It seeks to provide insights into what the issues and challenges are and what some potential strategies for addressing them might be.

In his paper on the three worlds of mathematical thinking David Tall describes a theoretical framework that enables us to examine and interpret the changing nature of the thinking that students experience as they move from the proceptual-symbolic world to the axiomatic-formal world of axioms, theorems and proof. The spiral, cyclical nature of the progress of mathematical thinking is well described, with school mathematics building from embodiment of physical conceptions and actions, through their symbolisation to manipulation of the symbols. These actions

in turn lead to formalisation of properties of objects as axioms, and logical deductions using these then lead to further higher-level embodiment and symbolism. However, Tall argues that the *met-befores* of embodiment and symbolism can have a subtle effect on a student's transition to the formal world, as met at university. This may play a role in one of the key distinctions between these worlds: the quality of the reasoning and argumentation employed. The informal, inductive approach to argument in school has to give way to formal, deductive proof, with a consequential change in warrant and backing. A number of these theoretical ideas are picked up by other authors later in the volume.

In contrast to Tall's psychological approach, the paper by Megan Clark and Miroslav Lovric proposes a framework that describes the school to university transition in mathematics as a modern-day rite of passage, involving separation, liminal and incorporation stages. Building on an anthropological models of rites of passage they explain that smooth transitions are neither to be expected nor are they necessarily desirable. They argue that transition takes time, and effort should not be expended in trying to speed it up. Furthermore students need to take responsibility for their learning. One interesting consequence of these principles is the proposal that at university it is "beneficial to expose entry-level university students to precise mathematical language and rigour of mathematical reasoning, and to insist on proper use of mathematical symbols and notation". The authors suggest that research analysing problems and issues in transition could usefully consider the dynamics in each of the separation, liminal and incorporation stages. This paper presents an interesting approach to transition in mathematics that will prove intellectually stimulating.

Gerd Brandell, Kirsti Hemmi and Hans Thunberg take a more traditional route in presenting evidence from the Swedish education system of a widening gap in curriculum and perceptions of mathematics between school and university. Data from new entrants to university in Sweden show a knowledge and skills gap that includes elementary functions, and certain inequalities and algebraic equations. Another area of attention is students' experience of proof. Since many students have not experienced a study of proofs, or practised producing them before entering university, it is not surprising that they have difficulty following them and constructing their own. The perceived widening of the gap is attributed primarily to a shifting emphasis in the secondary school curriculum without regard for university study, and the failure of universities to adjust their first year courses accordingly. There will be much in this paper that many readers will be able to relate to from their own experiences in other countries. The corresponding situation in South Africa is well documented by Johann Engelbrecht and Ansie Harding. Here they discuss the implications of a move from a skills-based to an outcomes-based education (OBE) system in High School on the transition to university, using data from a mathematics achievement test for three large groups of first year students. Interestingly the results show that in most content areas students under the new system performed on a par with previous years. However, there was a marked decline in performance on modelling and word sums and ratio and proportion, but a notable improvement in geometry. Hence, while the research suggests that there has been no major disruption to university preparedness from the OBE there are some areas of concern that warrant further investigation.

Two papers in this collection consider specific, related examples of mathematical content and the influence they may have in the transition from school to university. In the first David Godfrey and Mike Thomas consider the growth of thinking about equations, from an embodied output of the results of

procedures through the equivalence relation studied at university. Their study of student understanding at school and university suggests that the construction of a coherent conceptual image of equation is not straightforward for many students, with the input-output model of thinking persisting into university for around 25% of students. They suggest that because a formal world view of equality based on properties develops slowly across the transition from school to university it may be, in agreement with Clark and Lovric's position, that these properties should be explicitly and formally addressed in teaching, both at the upper end of secondary school and first year at university. The second paper, by Jarmila Novotna and Maureen Hoch, presents the results of an investigation of the relationship between structure sense in school algebra and university abstract algebra. Following a definition of structure sense in both of these contexts, they postulate, with examples, that high school structure sense is a subset of university algebra structure sense, and that the latter has components that act as analogies of the former. One conclusion from their data analysis is that explicit attention to structure in the symbolic world of high school could aid the transition to the formal world of thinking at university. The implication is that this should begin with number sense, followed by symbol sense and finally proceeding to structure sense.

It is one thing for research to identify problems in a field of study, such as the transition from school to university, but it is quite another to present realistic proposals for potential solutions. The paper by Talma Leviatan describes an attempt to address the gap between school and university mathematics by means of an innovative tertiary programme. The programme's rationale and objectives are documented along with details of the teaching methods. Some indications of the success of the programme from both student and lecturer perspectives are presented.

In the final paper in this collection Leigh Wood and Ian Solomonides apply a model of transition with separation, transition, and reincorporation phases to the move from school to university. This model has implications for those who teach at university, suggesting that they foster student engagement by addressing both cognitive and emotional variables. This applies equally at both university entry and exit points, with a need to enable effective transitions, including preparation of students for the workplace beyond university. Some excellent research-based suggestions on how this may be managed are put forward, including the possible imperative for academics not to look at a possible gap produced by the schools students come from, but to re-focus on where their students will progress to beyond university. This indeed presents a challenge for all involved.

I believe that this volume contains a broad collection of international papers with a range of excellent, and differing, insights into the increasingly important field of the transition from school to university mathematics, and beyond. I am pleased to have been involved in its production and am confident that the reader will find much that is intellectually stimulating and thought-provoking, as well as personally challenging in terms of practice, and I hope that it will encourage much discussion and further research in the field.

References

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