BOOK REVIEW

Understanding in Mathematics

Anna Sierpinska

London, The Falmer Press,
1994, 189pp.
ISBN: 0-7507-0334-2

Reviewed by Mary Coupland
University of Technology, Sydney

This book is the second in the *Studies in Mathematics Education* series edited by Paul Ernest. In five chapters and 189 pages, the author takes readers on a wide-ranging tour of the intellectual landscape called "understanding." For teachers of mathematics with some background reading in the philosophy and psychology of mathematics education this will be an enlightening tour over some familiar territory as well as interesting new ground.

In the introduction, Anna Sierpinska tells the story of her search in the literature of psychology, philosophy, education and linguistics for ways of understanding understanding. Bachelard's notion of Epistemological Obstacles seemed attractive, but this did not hold all the answers since her own experience showed that not all understanding consisted of overcoming obstacles. From Dewey came the idea that to understand meant to "grasp the meaning," but this was the opposite of Husserl's theory that defined meaning by understanding. To overcome this circularity, Sierpinska turned to Ajdukiewicz's (1974) writings on meaning and understanding in language and built on his definitions, adapting them from the field of understanding expressions in language to understanding mathematical concepts.

The first Chapter surveys the use of the term "understand" in everyday life and in the writings of Kotarbinski, Greeno, Bruner, Wittgenstein, Ricoeur, Poincare, Austin, Peirce, Frege, Ajdukiewicz, Hirsch and others. I found this chapter a little overwhelming, and the parts of it which stay with me are the illustrations of mathematics students and their struggles with certain mathematical ideas. Indeed throughout the book I found myself making most sense of the parts illustrated with examples from teaching experience.

By Chapter 2, Sierpinska is ready to build her own explanations and theory. Understanding is "an act of mentally relating the object of understanding to another object" (p. 28). The first object is called the "object of understanding," and the second, the "basis of understanding." Of course, it is necessary then to consider "object," and in order to avoid circularity, this is left as an undefined or primitive term.

But is this possible? Later, Sierpinska talks of something being an object "because we can isolate it as an object of our thinking, of our understanding" (p. 31). Mathematical objects are considered to be creations of the human mind (after Popper).

Since mathematical objects can be considered as being "embedded in a system of logical necessities and consequences of their relations with other mathematical objects, they may have properties that can be hard to discover, or difficult to prove or disprove" (p. 30). The mental operations involved in understanding are defined to be "identification, discrimination, generalisation, and synthesis" (p. 56). In expanding on this, Sierpinska has been able to draw on many established philosophical ideas as well as giving examples from the classroom. This is a strength of her writing, although a second reading is often needed to follow through the development of the main arguments since there is much embedded in the text that could, I feel, appear as footnotes or endnotes.

In addressing the psychological and social conditions of an act of understanding separately from and almost as afterthoughts to her chief definitions, Sierpinska seems to place herself in the first of the two "camps" of constructivists referred to by Paul Ernest (1994), namely those who espouse individualistic or cognitively-based theories, rather than socially-based theories. Yet, in Chapters 4 & 5 (which for me were the most interesting parts), an historico-cultural view arises which almost sees the tour leave for the opposite camp. By then, we have had, in Chapter 3, a look at processes of understanding, which "can be regarded as lattices of acts of understanding" (p. 72), and have been shown the author's perspective of deduction, inference, explanations, proof, figures of speech, activity, and the continuity of processes of understanding.

In the fourth Chapter, Sierpinska establishes that any judgement about a student's understanding will be relative to cultural norms. Using her definition of an act of understanding, it appears that the judgement about such an act depends on whether the basis of understanding conforms to an accepted or expected way of understanding the object of understanding.

The question of who does the accepting or expecting is not addressed, except to indicate that knowledge of the historical development of mathematical ideas would indicate to a teacher just which acts of understanding are more significant than others.

This naturally brings in the whole question of the relationship between philogenesis and ontogenesis, on which light is shed by considering the contribution made by Skarga to the tracing of past thoughts into present language, especially as metaphors. A very interesting description of the overcoming of epistemological obstacles in the development of the Bolzano Theorem helps the reader to gain a sense of how mathematicians build a theory by identifying errors or inconsistencies, and by stretching concepts. When concepts became inadequate, new ones were invented, such as the domain and range of a function.

While reading this, I wondered why so many textbooks leave out the history of such ideas or deal with history in small stories at the beginning or end of a chapter and then proceed with boring, sanitised definition after definition. I am sure that Sierpinska would want all lecturers to incorporate the historical side of mathematics into their lectures—not just a list of dates, but a larger picture of where the ideas came from—to provide the motivation to extend, unify, and solve. That these motivations are more than a "pure" devotion to the abstract advance of mathematics can be brought alive by readings in the sociology of mathematics: San Restivo, for example.

The final Chapter of the book uses examples from work with children learning mathematics to illustrate the Vygotskian view of the development of concepts. Sierpinska finds this useful for describing "the possible genetic forms of understanding" (p. 143), and the approach seems fruitful even for understanding how adults learn mathematics. Concepts cannot be *given* to a learner. Rather, concepts must be constructed from the learner's own experience which of necessity is with concrete things and (originally) small numbers. This is where unavoidable epistemological obstacles come from in the genetic sense. A learner faced with the task of making sense of the idea of infinity, for example, has to reorganise concepts and systems that have served very well in the (finite) past.

Culturally and historically, the same thing happens. Sierpinska uses Hall's (1981) model of a culture as a form of communication on three levels. This suggests that the informal level, at which understandings are unarticulated and formed in actions and thoughts, is the source of the more refined, rationally justified knowledge of the technical level. The problem is that we need the actions of the informal level to generate questions and hypotheses, but when we work on these at the technical level they bring with them concepts and schema which may be obstacles to good understanding.

On the whole I would recommend this book to those who are interested in the models that mathematics educators like to build about the process of learning, especially models with the flavour of continental philosophy. I was left, however, with a feeling of dissatisfaction about the way that social issues were acknowledged but not yet incorporated into the theory. For example, the recent work by Atweh and Cooper is described under a heading "The 'Social Handicaps' Influencing Mathematical Development" (p. 141), and the limiting nature of the school mathematics experiences of many children is described but not questioned. A theory of mathematical understanding which embraces the social construction of knowledge would go further towards explaining the motivations of those who choose not to participate in mathematical activity as well as those privileged who do.

References

Ajdukiewicz, K. (1974). *Pragmatic Logic.* Dortrecht: Reidel.

Ernest, P. (1994). What is social constructivism in the psychology of mathematics education? In J. P. da Ponte & J. F. Matos (Eds.), Proceedings of the 18th International Conference for the Psychology of Mathematics Education (Vol. 2 pp. 304-311). Lisbon, Portugal: International Group for the Psychology for Mathematics Education.

Hall, E. T. (1981). *The Silent Language.* New York: Anchor Press.