

RESPONSE TO BOOK REVIEW OF
Mathematics Education:
Models and Processes
 (English & Halford, 1995)

Challenges and Opportunities in Mathematics Education and Cognitive Science: A Reply to Collis

Lyn D. English and Graeme S. Halford
Queensland University of Technology *The University of Queensland*

Collis (1996) opens his review¹ of our book, *Mathematics Education: Models and Processes*, with the claim that it is aligned with the ICMI aims of the 1970s. He supports that claim by using three quotations from the book, all of which refer to linking mathematics education to cognitive science. The irony is that the cognitive science work which he uses as evidence for his claim did not exist in the 1970s.

Cognitive science as we now know it is much more a product of the 1980s and 1990s. The specific work which we use to provide a conceptual basis for mathematics education includes theory of analogy and mental models, parallel distributed processing (PDP) research, cognitive complexity theory, and planning and strategy development research, all of which developed in the 1980s and 1990s. Thus although Collis's review may be restricted to the 1970s, the book most certainly is not. If we want an objective indicator, the vast majority of the approximately 650 references in the book are from the 1980s or 1990s, whereas the five papers of his own work that Collis cites in his review

are from 1969 to 1976. Furthermore, the book is consistent with views expressed by Balacheff and others in the ICMI Study Series, *Mathematics and Cognition* (Nesher & Kilpatrick, 1990). In outlining future perspectives for research in the psychology of mathematics education, Balacheff stated:

Research on the learning of algebra, geometry, or calculus cannot develop without a deep epistemological analysis of what the concepts considered consist of as mathematical concepts. Also, it is recognized that the meaning of mathematical concepts relies not only on their formal definition but more fundamentally on the processes involved in their functioning. It is for this reason that emphasis is put on the study of students' cognitive processes rather than on their skills or actual productions. (p. 136)

Collis does not mention our discussion of Fischbein's (1990) call, in the same ICMI Study Series, for a collective endeavour to develop a theoretical background for the mathematics education field. To quote part of our statement of aims:

We agree with Fischbein that mathematics education generates its own set of psychological problems that are foreign to the professional psychologist. The countless number of misconceptions evident in children's mathematical performance in school cannot be solved by simply applying some general psychological principles. We need to also address theories of mathematics curriculum development. A marriage of these two domains should go a considerable distance towards producing an effective psychological theory of mathematics education. Our book aims to develop such a theory. (English & Halford, 1995, p. 18).

1. The review was published in November 1996 in the *Mathematics Education Research Journal*, 8(2), 198-202.

After outlining approaches to the field (Chapter 1) and relevant concepts from cognition (Chapter 2), the book addresses the question of how mathematics is understood (Chapter 3). Research in cognitive science has shown that human understanding is based on mental models which are content-specific and analogical in character. Thus mathematical operations are first understood by analogies based on physical operations on objects, while algebraic ideas may be understood by using the structure of arithmetic operations as analogs. This research yields explanations for why some concrete teaching aids, which are technically analogs, are effective and others are not. Good teachers recognise that teaching aids are not panaceas and need to be chosen thoughtfully, and we have intentionally linked the best insights of practitioners with the research of cognitive science. Analyses of mathematical analogs leads on to complexity theory, which further explains why some concepts seem unaccountably difficult while others are not. Again, competent teachers often sense these facts, but they lack clear explanations. Appropriate analyses of the relevant cognitive processes provide more rigorous explanations, which lead to more effective remediation. Collis' review has simply missed all of this.

To illustrate, we have shown how students' recognition of the correspondence between mathematical concepts, one of the most fundamental goals of mathematics learning, depends on mapping common structures (relations) from one concept to another. Structure-preserving mappings from one concept to another is what analogy is all about (English, 1997; Gentner, 1989; Halford, 1993) and analogy theory has permitted deeper insights into

the causes of fragmented knowledge. We did not simply transport analogy, or any other theory, from its original contexts, but developed and extended it to account for problems in mathematics education. Clear understanding of how links are made cognitively between concepts can help to discriminate between good and bad instructional procedures, which may have been indistinguishable otherwise. Good practitioners may already be using some of these procedures and their intuitions are, themselves, a valuable source of insight. However even the best practitioners do not agree on which approaches are most effective. Analyses of the cognitive processes involved in mathematics learning can do a lot to sharpen the discriminations of practitioners, who, in turn, can challenge and extend the work of cognitive scientists. Our work aims to bring cognitive scientists and practitioners of mathematics education into a partnership for this purpose. We believe our book helps to indicate how productive this partnership can be.

While we acknowledge the importance of sociocultural and other related factors, both within and beyond the classroom, the analysis of mathematical concepts and how these concepts map into relevant cognitive processes is essential to the development of more effective mathematics education. We are happy to acknowledge the contributions to this effort by Collis and others in the 1970s, but most of the exciting developments in cognitive science have occurred since that time, and our book utilises those developments to offer a new approach to longstanding problems. One of us (English) has a background in mathematics education, including the very practical and demanding process of writing curricu-

lum materials for children in remote Queensland locations without access to formal schooling. The other (Halford) has a background in cognitive science. While writing the book, we were both surprised and gratified to see how readily we could synthesise our areas of expertise to produce a coherent account of mathematics education. Perhaps we underestimated how much of a challenge this approach would be to some earlier contributors to the field, but we would invite the reader to see for her/himself whether we have offered anything of value. The contribution certainly seems to have been recognised in Russia, judging by a recent review (Poddiakov, 1996). We are confident that readers elsewhere who are receptive to new approaches with both conceptual power and practical utility will be rewarded.

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