Noticing Mathematical Pattern and Structure Embodied in Young Children's Play

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This position paper proposes that a relationship between young children's embodied mathematical concepts and their awareness of mathematical pattern and structure (AMPS) (Mulligan & Mitchelmore, 2009) develops through play. Theoretical perspectives on the development of schematic patterns, the embodiment of mathematical understandings, and the development of AMPS are outlined. We propose AMPS may underlie children's embodied actions in play. Thus, the practice of professionals' 'noticing' is central to supporting children's development of mathematical concepts. Implications for further research, including the development of an observational framework to notice AMPS through play, are discussed.

Young children may reveal their mathematical ideas naturally through play, providing a valuable context for early childhood educators to respond authentically to children's mathematical curiosities. However, educators' awareness of the importance of interpreting mathematical possibilities, and their perceived lack of confidence and corresponding mathematical content knowledge can inhibit their response to these playful encounters (Cohrssen, 2015; Lee, 2107). Early Childhood Australia (ECA) recognises the difficulty some educators experience in responding to mathematical concepts young children engage with, acknowledging that many feel "less comfortable having conversations with children that enable [them] to assess their mathematical thinking during play" (Cohrssen, 2015). However, intentional observational practice is advocated in the Early Years Learning Framework [EYLF] (Department for Education, Employment and Workplace Relations [DEEWR], 2009). Thus, there is a need to support educators to notice the complexity of mathematical concepts children are naturally exploring, and to actively engage in questioning and dialogue to elicit children's reasoning to inform future directions for learning (Cohrssen, 2015).

Children's natural ability to notice, develop and utilise mathematical ideas to make meaning of experiences has been well documented through the practice of observing their play (Marcus, Perry, Dockett & MacDonald, 2016). This research found that young children "noticed … explored … [and] talked about the mathematics they encountered … highlighting the importance of conversations with children" (Marcus et al. 2016, pp. 441; 445). Play has been widely recognised as an enabling context whereby the breadth and depth of children's true competencies can be observed (Rinaldi 2013; Van Hoorn, Nourot, Scales & Alward, 2015). Through play, young children's mathematical understandings can be interpreted as an embodied representation of their thinking (Thom, 2017). Research into embodied cognition recognises the role of physical movement and playful engagement with learning environments as critical factors in the formation of preverbal mathematical

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concepts (Kim, Roth & Thom, 2010; Nunez, Edwards, Matos, 1999). Bautista, Roth and Thom (2012) recognise that movement alone reveals thinking, proposing that there is a relationship between movement and “the emergence of abstract mathematical knowledge” (p. 363). The authors contend that abstraction of mathematical understanding is the result of engaging physically with “rhythmic patterns [that] emerge in corporeal-kinetic dimensions” (Bautista et al, 2012, p. 368), concluding that these cognitive structures are a consequence of engaging physically with the world. For example, as children move around boundaries of play spaces, they experience this as an iteration of steps, and larger spaces are experienced as requiring more paces. Movement patterns such as this naturally emerge from young children’s play as they repeat and iterate known actions, ‘schemes’ that reveal “organised patterns of behaviour” (Piaget, 1952, cited in Ginsburg & Opper, 1969, pp. 20-21). Athey (2007) identified eight common dynamic behavioural patterns, termed ‘action schemas’. These observable actions have been likened to the development of “conceptual clusters … [cognitive systems] that illustrate aspects of spatial thought” (Hayes, 1979, cited in Athey, 2007, p. 2). Therefore, the ways that children interpret and engage with spatial features of play spaces can be analysed through observing their pattern of actions (Athey, 2007).

Pattern and structure, are widely recognised as foundational in the development of mathematical understandings in the early years (Mulligan & Mitchelmore, 2009; Papic, Mulligan & Mitchelmore, 2011). Specifically, an awareness of mathematical pattern and structure (AMPS) (Mulligan et al., 2009) has been found to be critical in the development of pre-algebraic reasoning, supporting the abstraction and generalisation of mathematical concepts (Kieran, Pang, Schifter & Ng, 2016; Mulligan & Mitchelmore, 2013; Papic et al., 2011). For example, young children reveal an awareness of mathematical pattern and structure when they notice similarities between objects; when they can recognise what is the ‘same’ across a variety of experiences; or predict what may come next in a sequence of events, reasoning and predicting about change. Mathematical thinking emerges from all aspects of life (Ernest, 1991, 1994), therefore an awareness of pattern and structure could be considered to underlie life experiences (McCluskey, Mulligan & Mitchelmore, 2013). Thus, an implication for early childhood education is that the development of mathematical pattern and structure could be observable in the children’s embodied actions.

The intention of this position paper is to provide a rationale for exploring the relationship between the embodiment of mathematical understandings and the development of children’s awareness of mathematical pattern and structure. Theoretical perspectives into the development of schematic patterns, the embodiment of mathematical understandings, and children’s awareness of mathematical pattern and structure are presented to propose that an awareness of mathematical pattern and structure may underlie children’s play. The development of educators’ pedagogical content knowledge and the notion of active noticing will also be discussed. Two research questions are raised; does AMPS underlie young children’s embodied actions in play? and, what effective elements of practice could support educators’ active noticing of children’s AMPS through play?

Background

Viewing the Child as Mathematically Competent

Early Childhood Australia’s (ECA) *Strategic Plan 2014-2017* identifies the need to realise each child’s innate capacity to grow and learn, advocating for children in all Australian contexts to be viewed as already capable and competent (ECA, 2014). This has implications for practice whereby educators' focus becomes geared towards observing what children can do rather than noticing what they can't do (ECA, 2012). A strength-based approach advocates for early childhood practitioners' to notice and base future learning directions upon children's current capabilities (Connor, 2011; Department of Education and Early Child Development [DEECD], 2012; ECA, 2012). Thus, assessment practices that capture, document and reflect children's existing 'thinking in action' is an integral aspect of ongoing planning for learning in early childhood (Cohrssen, 2015).

This view of the child as mathematically capable is iterated in the *Early Years Learning Framework [EYLF*], (Department of Employment, Education and Workplace Relations [DEEWR], 2009); this national document guides the practice of early childhood educators, who work with children from birth to 5 years of age across Australia. The EYLF acknowledges that "all children demonstrate their learning in different ways …[therefore] … approaches to assessment are … relevant and responsive to the physical and intellectual capabilities of each child", and these observations are connected to learning outcomes (DEEWR, 2009, p.17). However, the broadly stated outcomes in the EYLF provide minimal reference to the specific type and depth of mathematical thinking, processes and concepts children engage with. For example, outcomes 4.2 and 4.3, draw attention to children "communicating mathematical ideas and concepts; using patterns, mathematical language and symbols; contributing constructively to mathematical discussions and arguments; making connections; solving problems; applying generalisations; trying out strategies; and transferring knowledge" (DEEWR, 2009, pp. 35-36). Thus, specific information, regarding how mathematical ideas and concepts transform as children develop greater conceptual awareness, is not evident or elaborated upon in the EYLF. Similarly, identifying the possible depth and range of young children's mathematical thinking is not well supported through the descriptions and analysis of learning stories in the accompanying resource, *Educators: Belonging Being & Becoming* (DEEWR, 2010).

Educators' Noticing of Mathematical Features of Children's Play

The Department of Education and Child Development (DECD) in South Australia has produced *Implementation guidelines for indicators of preschool numeracy and literacy* to support early childhood educators identifying mathematical features of children's interactions (DECD, 2015). This document refers to a *Numeracy Chart* that is multi-layered, whereby mathematical processes (behavioural) and four broad indicators (conceptual) articulate how a "child sees, interacts with and explores their world" [mathematically], the indicators are "interconnected and observable," relating broadly to young children's developing mathematical senses (DECD, 2015, p. 9). Key elements and examples of practice in the document support professional learning and further discourse around identifying children's mathematical thinking consequently strengthening educators' pedagogical content knowledge. However, the children's thinking in the documented examples is not captured over time, presented as isolated exemplars. Thus, the development of mathematical understanding, and pathways that delineate changes in awareness of specific mathematical concepts, is not revealed.

In their *Evidence Pape*r on *Assessment for Learning and Development* in early childhood contexts, Flottman, Stewart and Tayler (2011) state that "non-judgemental assessments [that are] evidence based … dynamic and ongoing …systematic and rigorous" (p. 5), are essential elements in effective assessment for young children's learning through play. Ongoing, formative assessment strategies underlie the intentional, reflective practice affirmed in the EYLF (DEEWR, 2009; DEEWR, 2010). However, there is a need to support educators to notice the complexity of mathematical concepts children are naturally exploring, and to actively engage in questioning to elicit children's reasoning. "Early numeracy skills predict [later] achievement in mathematics … [and] … greater growth in numeracy skills [is] related to greater maths-specific talk amongst teachers" (Reid, 2016, p. 7). Therefore, the development of systematic, evidence-based practices and professional learning resources are needed to support early childhood educators' noticing and responding to the depth of young children's mathematical thinking (Lee, 2017).

Theoretical Perspectives

This paper considers three theoretical perspectives-theories concerning schematic patterns, embodied cognition, and the awareness of mathematical pattern and structure [AMPS] (Mulligan et al., 2009), to explain the development of children's mathematical understandings revealed through their embodied actions in play. In presenting these perspectives we propose that an inherent relationship exists between them, which then leads tentatively to the formation of a multi-dimensional theoretical framework. Approaches to supporting professional learning to develop educators' pedagogical content knowledge and active noticing of children's mathematical understandings will also be discussed.

Action Schemas, Embodied Cognition, and Early Childhood Mathematics Education

Piaget asserts that young children display "order and coherence" through the development of patterned schemes, which "refer to the basic structure underlying the child's overt actions … [there is] structure of behaviour; that is an abstraction of the features common to a variety of acts which differ in detail … [observable as a] regularity of behaviour" (Piaget, 1952, cited in Ginsburg et al., 1969, pp 20-21). Chris Athey (2007) built upon Piaget's theories regarding common patterns of behaviour, categorising these into eight observable action schemas that relate to how children engage spatially and naturally within their environment. These being; "dynamic vertical; dynamic back and forth; circular rotation; going over, under or on top; round a boundary; enveloping or containing; going through a boundary; and [externalised] thought" (Athey, 2007, p. 3). The schemas progress through four distinct but interrelated stages, "sensorimotor behaviour, symbolic representation, functional development, to thought", interestingly the final stage, the expression of thought, which is also one of the action schemas, emerges from a coordination of the other action patterns (Athey, 2007, p. 2). However, the structural elements underlying the development of the action schemas at the sensorimotor level requires deeper understanding to interpret 'order and coherence' underlying these behavioural patterns and to connect this with the perspective of embodied cognition.

Thought is not always expressed verbally; non-verbal actions are readily observable through children's play and can reveal children's patterns of thinking mathematically (Athey, 2007; Ginsburg et al., 1969; Kim et al., 2010; Piaget, 1926, 1952). Children's expression of mathematical concepts, initially formed as a bodily sense of knowing, have been studied through the field of embodied cognition (Kim, et al., 2010; Meltzoff, 1999; Merleau-Ponty, 2002). Underlying the theory behind embodiment is the premise that bodily intelligence emerges through the child's movement and sensory engagement with environments as "movement is the mother tongue," the first outward expression children engage with, from which all other cognitions are derived (Smith & Gasser, 2005, p. 29). Bautista, Roth and Thom (2012) "propose that kinetic movement constitutes thinking itself," referring to dynamic action as "thinking in movement" (p. 380), whereby mathematical concepts are "in the flesh" (p. 364) and thus experienced through the movement of the body. Observable features of children's dynamic movement include actions such as "rhythmic patterns … beat gestures … body position and object orientation" (Bautista, et al., 2012, p. 368). Therefore, children's movement in action, iterated over time would reveal patterns of commonalities, emerging across experiences; and these similarities could reveal an underlying coordinated structure (Athey 2007; Ginsburg et al., 1969; Piaget, 1952).

Children's Development of an Awareness of Mathematical Pattern and Structure (AMPS)

Having a deep, fluent understanding of how mathematical concepts develop requires educators to have an awareness of the structure of concepts observed (Mason, Stephens & Watson, 2009). Structural understanding involves reasoning about the relationships between patterns to recognise and engage with similarities between and across concepts (Wood, 2002). Patterns have predictable elements that repeat (Mulligan et al., 2009). Thus, reasoning about the relationships between the repeating elements, and across different types of patterns, leads to generalisations about structural features underlying all mathematical concepts (Mason et al., 2009; Mulligan et al., 2009). Mulligan, Mitchelmore and Stephanou (2015) identified these underlying mathematical structures as: sequences; shape and alignment; equal spacing; structured counting; and partitioning. An interview-based Pattern and Structure Assessment [PASA] (Mulligan et al., 2015) can be implemented to measure individual children's levels of AMPS and underlying structural development. Longitudinal research with children aged 5 to 7 years has found that attention to these structural features of mathematical concepts supports the growth in children's level of awareness of mathematical pattern and structure [AMPS] (Mulligan et al., 2015). However, formal methods of assessing children's understandings can remove children from contexts and experiences that are familiar and meaningful to them (Cohrssen, 2015; Macmillan, 2009; Papic, 2015; Van Hoorn et al., 2015) and thus may not indicate the depth of children's existing mathematical capabilities revealed through familiar contexts such as play.

Professional Learning Perspectives: Developing Early Childhood Educators' Practice of Noticing Mathematical Features of Children's Play

There is an expressed need to support educators' awareness of young children's embodiment of mathematical thinking (Thom, 2017). In a recent reconceptualization of early childhood educators' mathematical pedagogical content knowledge (PCK), three essential underlying constructs were identified: '*noticing'* everyday mathematical opportunities; '*interpreting'* the mathematical content inherent in these situations; and '*enhancing'* this to deepen the children's mathematical thinking (Lee, 2017, pp. 232-233). These three interrelated aspects of PCK are developed through educators' "knowledge of children's development [of] mathematical concepts as well as their own knowledge of strategies … [and] are effective only when teachers have a depth of personal understanding about mathematical content … to mathematize children's informal experiences" (Lee, 2017, p. 241). In Lee's study (2017) it was found that the educators' ability to notice geometric/spatial elements of the children's play was substantially less than noticing aspects of number/measurement, and that "ability to notice [did] not necessarily translate into effective execution of interpretation" (p. 240). Strengthening educators' mathematical knowledge was identified as key in supporting the development of all three interrelated aspects of PCK, that is: noticing, interpreting and enhancing young children's mathematical thinking (Lee, 2017).

Summary and Recommendations

This paper presents a theoretical proposition that an awareness of mathematical pattern and structure [AMPS] (Mulligan et al., 2009) could underlie the development of children's embodied mathematical cognition. Thus, identifying children's use of, or attention to, pattern and structure evidenced from observing their dynamic movement through play could provide a more integrated view of their developing mathematical understandings. As this relationship is yet to be fully described, the development of a multi-dimensional observational framework is planned. The aim is to focus educators' attention to noticing children's pattern of actions (Athey, 2007). This will be overlayed with the five mathematical structures indicating levels of AMPS (Mulligan et al., 2015). This could reveal insight into how to examine the emergence of AMPS (Mulligan et al., 2009) underlying embodied actions in children's play.

Educators' practice of interpreting children's developing mathematical understanding through noticing patterns of dynamic movement expressed in their play has important implications for early childhood mathematics education (Lee, 2017). Interpreting embodied expressions is reliant upon educators' own awareness of recognising young children's mathematical thinking as concepts in action and knowing how to respond to differing levels of children's awareness. This will enable educators to connect with and develop these non-verbal understandings (Cohrssen, 2015; Lee, 2017; Marcus et al., 2016; Thom, 2017). Therefore, further research into the design of professional learning that supports educators' PCK may explore how pattern and structure could underlie children's embodied action, revealing a more coherent picture of young children's mathematical development noticed through play.

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