Place-based mathematics and Projection Augmented Landscape Models (PALM) on the Tiwi Islands: Building Powerful Aboriginal Hypothetical Learning Trajectory

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This paper presents an account of a new place-based education initiative using both the mathematical construct of hypothetical learning trajectory and the 3D printing technology or Projection Augmented Landscape Models (PALM), as a tool for learning. The goal of the pilot study is to create bridges between Aboriginal and Western ways of knowing and learning to enable academic success in the challenging area of mathematics. The project explores powerful ways to construct cultural learning trajectory in which Tiwi preservice teachers can apply their prior traditional knowledge to their learning and teaching of Western mathematical constructs, within the Australian curriculum strand of geometry/measurement. The outcomes of this unique Tiwi project are significant because it is an applied research project using 3D PALM technology for educational purposes, particularly knowledge exchange in Aboriginal contexts. This application is unique as such technology to date has been used largely in applications related to land management.

Background and Statement of the Problem

The School of Education at Charles Darwin University (CDU), in partnership with the Northern Territory Catholic Education Office, has developed a community-based teacher education program, *Growing Our Own (GOO)*, enabling Aboriginal Teacher Assistants currently working in remote schools to gain a relevant teaching qualification (Giles, 2010). Where mainstream teacher education students access courses externally via the Internet and/or internally by attending lectures and tutorials, the GOO program enables Tiwi preservice teachers to access the course in situ, with the lecturer traveling to Tiwi Islands. Online learning outside of the school is challenging for the preservice teachers as Internet access is intermittent on Tiwi Islands and not available in most of their homes. The GOO program seeks to overcome some significant barriers for students who would otherwise have little or no chance of becoming qualified teachers, not from lack of ability but from accessibility.

This pilot study explores ways in which Aboriginal preservice teachers can apply their prior traditional knowledge of the local Tiwi context to their learning and teaching of the Western mathematical construct of hypothetical learning trajectory using *Projection Augmented Landscape Model (PALM)* as a technological tool. The learning trajectory is 'the' construct par excellence of the unit EMA200 [Mathematics Education 2: Curriculum and Pedagogy in Primary Mathematics] at CDU and includes a series of face-to-face workshops to be delivered in August 2019. This concept, described below, will provide some guidance for the Aboriginal Tiwi preservice teachers in mathematics instruction. The goal of the project is to support Aboriginal preservice teachers to culturally and meaningfully create powerful mathematics learning trajectories by creating inroads between Tiwi and Western

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ways of knowing and learning to enable academic success in the challenging area of mathematics. We are introducing the pilot study as part of a new initiative that aims to bridge Australian mathematics curriculum content descriptors of the measurement strand with the place-based Aboriginal Tiwi context and local Tiwi language using the 3D printing technology (PALM). In other words, this study promises to make mathematics culturally relevant using a cognitive tool (learning trajectory) to generate a Tiwi 'textbook' from the PALM technology (local stories and language place-based) to teach mathematics from. The study is a meaningful and localised mathematics solution for the Tiwi preservice teachers; a bridge between the local Tiwi language and culture and the Western subject mathematics.

Hypothetical Learning Trajectory

The hypothetical learning trajectory is a concept that has gained some currency in the research literature in recent years. Battista (2011) describes a learning trajectory as "a detailed description of the sequence of thoughts, ways of reasoning, and strategies that a student employs while involved in learning a topic, including specification of how the student deals with all instructional tasks and social interactions during this sequence" (Battista, 2011, p. 511). Although an "actual learning trajectory is not knowable in advance" (Simon, 1995, p. 135), therefore hypothetical, teachers can use the concept of a learning trajectory in their planning. The learning trajectories are not linear and simple, but complex and branchy. Sarama and Clements (2009) claim that a mathematical learning trajectory is "a useful tool in understanding and supporting the development of children's mathematical reasoning" (p. 63). Stephens and Armanto (2011) advise that constructing a powerful learning trajectory simply means that it is "designed so that all students can engage with the problems presented to them" (p. 529). In his seminal work, Simon (1995) claims that three ingredients must be included in the hypothetical learning trajectory: "the learning goal, the learning activities and the thinking and learning in which the students might engage" (p. 133):

1) The Tiwi preservice teachers will set *learning goals* that need to be suitable for the age and cultural background of primary students. The Assignment 1 goals for preservice teachers' learning trajectory will originate from the strand Geometry and Measurements (Year 6) of The Australian curriculum. The content descriptions are for the most part outcomes which describe the outward manifestations of students' learning, but which say little about the students' thinking and process of inquiry.

2) *The sequences in mathematics learning* exist as pathways to achieve the teacher's goals. However, these progressions can vary with experiences, culture and for individual learners. For instance, in the EMA200 unit prescribed textbook (Siemon, Beswick, Brady, Clark, Faragher & Warren, 2015, pp. 30-31), Jerome Bruner's developmental progressions or three stages model reflects the thinking and learning that accompany the mathematics tasks [Enactive stage (concrete materials), iconic (diagram + visual), and symbolic representations].

3) The *mathematical tasks* or learning activities to achieve the goal of the trajectory need to be suitable for rich learning; that is to say, tasks need to be varied, engaging (constructivist) and collaborative, progressing from comparatively unstructured explorations of a concept to more systematic and organised representations.

The 3D PALM Technology and Application: NT and International Engagement

PALM has been used since 2016 for participatory planning activities when spatial datasets and fire spread simulations are displayed as a projection onto 3D printed landscapes or sand that is shaped to fit the terrain (Fisher, 2018). The PALM technology won the *Nation*

Education Technology award 2018 for Indigenous engagement with fire management as an application of the 3D model. It has resulted in significantly enhanced outcomes through supporting cross-cultural communication and leading to more inclusive learning in Arnhem Land (Fisher, 2018a). The PALM model has also had international engagement with fire management in Mexico, Indonesia and more recently in Cameroon for a range of environmental planning applications (Fisher, 2018). The 3D PALM model of the Tiwi Islands created from a satellite image is a new mathematics education initiative. The large 3D landscape prints are produced as multiple tiles (see Figure 1) that will be provided to students as a landscape jig-saw puzzle for re-construction which will further promote physical engagement. This PALM model has not yet been investigated pedagogically as a mathematics teaching tool in classrooms with preservice teachers and therefore very significant.

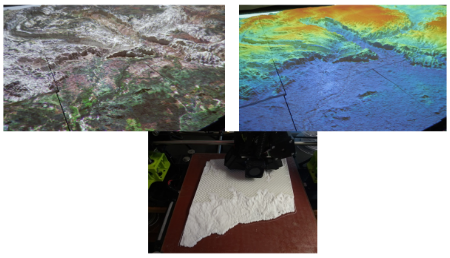

Figure 1. PALM jig-saw puzzle and printing prototype tile of the Tiwi Islands.

Discussion of Significance

A large body of mathematics research has shown the benefits of multisensory engagement significantly improves learning outcomes (Elwood, 2011; McCall & Dunn, 2012). Mathematics education that employs unisensory stimulus regimes does not engage multisensory learning style of Indigenous People who from an oral culture are hands-on practical learners (Shams & Seitz, 2008; Yildirim & Jacobs, 2012). Traditional mathematics education is thus a barrier to learning for Indigenous learners. If we want to enable success then we, as educators and curriculum developers, need to attend to methodological approaches that engage Indigenous learners in culturally relevant ways (Hogue, 2018). Incorporating PALM technology to create Tiwi landscapes as a 'textbook' into the teaching of mathematics will go a long way to engaging Tiwi learners.

In the Tiwi context, preservice teachers will have the opportunity to think deeply about the relationships between the construction of their Tiwi learning trajectory, their ways of knowing, the 3D PALM technology as well as their selected Australian curriculum mathematics goal. The following content descriptors and corresponding elaborations could be used (Year 6, Australian Curriculum, Assessment and Reporting Authority-ACARA, 2014); *Connect decimal representations to the metric system (ACMMG135)* [recognising the equivalence of measurements such as 1.25 metres and 125 centimetres]. The 3D model invites preservice teachers (children) to engage because it is aesthetically pleasing to the eyes. This is significant! First a comparative view of advantages between a 2D topographic map and 3D PALM mapping with cultural overlays is useful for preservice teachers. Imagine… a digital projector on the 3D map surface and an infra-red light pen to point to certain regions of the Tiwi Islands. Imagine… using layers of colours or cultural overlays inviting learners to storytelling, Tiwi language and culture, progressively leading to measurement outcomes from the Australian curriculum in each and every preservice teacher's learning trajectory: green may represent mangrove vegetation or medicinal plants on Tiwi Islands, black for roads, red for specific animal locations, dark blue for Tiwi waterways, brown overlay for ceremonial grounds or sacred sites, orange for locally iconic Pandanus tree range (*Pandanus spiralis*) alongside particular creeks, etc. Imagine a variety of rich localised tasks related to the study of measurement to include inside their cultural learning trajectory. Imagine their local perspectives and ways of viewing and understanding nature, ways of living weaved in and out from both Western and Tiwi knowledge traditions. How many kilometres between the two sacred sites? How many metres between the creek and the café and what is the equivalence of measurements in kilometres?

Learning Trajectory Tool + PALM Tool: Constructivism

The hypothetical learning trajectory is constructivist in that learning is seen as accompanying tasks as well as the interaction around them. Clements and Sarama (2004) claims that the "trajectory construct is a cognitive tool grounded in constructivism" (p. 85) and differs from other models in that it involves self-reflexive constructivism. Siemon et al., (2015) affirm that implicit to constructivism is "the importance of the learner's existing knowledge, constructed on the basis of past experiences. Each learner's experience is unique, and hence current knowledge means that each learner is likely to construct differing knowledge in response to a given experience" (p. 34). Ultanir (2012) states that "students construct meanings for concepts. As a result, learning is best undertaken in real world contexts in which students may acquire and test concepts" (p. 205).

The PALM model has the potential to engage Tiwi preservice teachers 'constructively' in their preparation of a culturally meaningful hypothetical learning trajectory with many mathematics lessons. The 'cultural Tiwi textbook' or 3D printed landscape model of the Tiwi Islands with projected spatial data (cultural overlays) makes the digital tactile, and through the additional infra-red sensor technology, the printed surface itself becomes interactive. We believe that the constructivist PALM technological tool, accompanying the learning trajectory, is highly significant for Tiwi Islands' preservice teachers because this 'local textbook' is a powerful hands-on 3D tool for all mathematics classrooms on Bathurst Island or for carrying away out in the bush with children. This study will develop Tiwi constructivist views of teaching and learning where the learner occupies the top position rather than the teacher/lecturer.

Critical Analysis of the Research Literature

Watson and Mason (2006) critique the model of the learning trajectory, saying that it appears to assume predictable and sequential development. They contend that learning is less predictable and more diverse than the learning trajectory model suggests. However, they acknowledge that "when planning, thinking in terms of hypothetical learning trajectories may contribute to teacher-confidence, giving them at least a place to start" (p. 97). To a certain extent, a trajectory involves an implicit sense of possible experiences that it is to be hoped each learner will transform into a personal learning trajectory (Watson & Mason, 2006). After all, teaching takes place in sequential time, even though learners take different trajectories.

Stephens and Armanto (2011) affirm that the idea is not to have longer and heavier mathematics textbooks in the primary years but simply to have carefully chosen examples and a well-developed learning and teaching trajectory. In this study, the rich Tiwi tasks attached to the learning trajectory will be selected using the 3D PALM technology and puzzle pieces and we believe these to be rich enough for every preservice teacher to engage and develop conceptually strong key measurement ideas from the Australian curriculum that will promote deep mathematical cultural thinking.

The success of Aboriginal students at tertiary level is related to culture (Hogue & Forrest, 2018). Aboriginal knowledge is valuable and does expand the definition of mathematics so that it can include Aboriginal knowledges in a respectful way (Michie, 2015). Sterenberg and Hogue (2011) claim that the approach of intertwining Aboriginal and Western knowledge systems is respectful and holds generative possibilities for mathematics education. We believe that listening and understanding the perspectives of Tiwi Islanders preservice teachers in mathematics is a powerful route for constructing engaging cultural learning trajectory in mathematics and for supporting the development of English literacy.

Language and Mathematics

English literacy is a crucial factor in the learning of mathematics (Edmonds-Wathen, 2014). For preservice teachers the GOO program, the most difficult idea in teaching mathematics is the students' ability in Standard Australian English (Thornton, Giles, Prescott & Rhodes, 2011). The 3D PALM tool provides an avenue for the preservice teachers to relate Western mathematics (hypothetical learning trajectory and measurement content descriptors) to their authentic context, reality and local experiences. However, there are language challenges for GOO mathematics lecturers visiting remote communities. The gap between Indigenous languages (Tiwi language in this pilot study) and formal mathematical constructs of the Australian curriculum is as much of a challenge for the GOO preservice teachers as for the Tiwi children. This is an issue requiring much attention for lecturers (Thornton, et al., 2011). The way that mathematics is conceptualised and described in the Australian curriculum may be different to the way it is talked about by preservice teachers. Jorgensen and Lowrie (2018) acknowledge the cultural context/s within which we work. Preservice teachers in "remote contexts speak a home language (or many languages) that are not the language of instruction. The mathematics vocabularies of the home and school may be quite different, so consideration needs to be made of this difference" (p. 441). Place-based PALM accommodation of preservice teachers' English literacy needs and oral stories are envisioned in the writing of their Tiwi learning trajectories.

Cultural Responsive Pedagogies: Place-Based Mathematics and Oral Stories

Culturally responsive pedagogies in mathematics education using 'place' and 'local stories' are gaining momentum with Aboriginal preservice teachers around the world. Smith and Rioux (in press) and Aikenhead and Michell (2011) contend that recognition of place-based knowledge and contexts provide an opportunity for the coexistence of Aboriginal ways of knowing and the Western-based school mathematics curriculum. "The significance of place is becoming a predominant theme in Aboriginal mathematics education", according to Sutherland and Swayze (2013, p. 179). Barnhardt and Kawagley (2008) advocate that successful education, curriculum and programming for Aboriginal people translate to a Pedagogy of Place or learning through culture. Such a view of place and culture provides a pedagogical discourse that moves "from teaching about local culture to teaching through the culture as students learn about the immediate places they inhabit and their connection to the larger world within which they will make a life for themselves" (p. 113). We argue for an education in tertiary institutions to be relevant and not dominated by pedagogical devices devoid of the place or local environmental knowledge.

The construction of cultural learning trajectories requires a place-based approach and strong pedagogical strategies. Sarama and Clements (2009) suggest understanding, using learning trajectories and including a wide variety of instructional tasks and teaching strategies. They recommend "using a combination of teaching strategies" (p. 64): Discuss a problem with a group, followed by children to work in pairs; Have the children share solution strategies back with the group; Discuss strategies with children in pairs and individually; and Differentiate instruction by giving groups or individual children different problem types. Additionally, we value a 'dialogical interaction' approach to Aboriginal mathematics education as an imperative in addressing the issues of cultural differences faced by Aboriginal students at tertiary institutions, between Western mathematics academia and local Aboriginal knowledge traditions (Sterenberg and Hogue, 2011). The Tiwi preservice teachers will generate local oral stories for the trajectory by working with a 'cultural textbook' [the 3D terrain surface (PALM)] as this will help with understanding measurement of distances within the local Tiwi place. As one Arnhem Land Aboriginal ranger said about PALM: *"everyone shared knowledge, both Yolngu and Balanda…. not shy"* (Fisher, 2018a).

The construction of cultural learning trajectories requires a place-based approach, strong pedagogical strategies and oral stories. Oral stories have been the text of Aboriginal culture worldwide. Bessarab and Ng'andu (2010) discuss research and storytelling as "doing research that takes into account culturally appropriate processes to engage with Indigenous groups and individuals is particularly pertinent in today's research environment" (p. 37). "Story telling is a feature of Indigenous societies where oral traditions are the main form of transmitting and sharing knowledge with individuals and between groups" (p. 38). For Hogue (2018), "narrative is one of the most powerful teaching tools and ways of relating experience. The transformative effects of listening and telling, and the ability to reveal aspects of human experience are some of the strengths of narrative as text" (p. 31). The cultural learning trajectory and PALM technology will rekindle transgenerational knowledge transmission via storytelling of the land with children, parents, grand-parents, and/or Elders. The appreciation of local stories of the land serves as a bridge between learning trajectory and Western mathematics for local preservice teachers so a clash of cultures can be addressed (Thornton et al., 2011).

Conclusions and Implications for Mathematics Education

CDU and the GOO program are well positioned to collaborate more closely with schools on Tiwi Islands and increase the preservice teachers' confidence to address any concerns relating to teaching, learning and understanding mathematics. The idea of building powerful Tiwi hypothetical learning trajectories via the creation of a culturally relevant 'textbook' (PALM model) is a fruitful way of looking at measurement descriptors of the curriculum in the primary years, because the PALM model is like a meaningful animated hologram of country. The 3D physical landscape jig-saw model (PALM) will support the preservice teachers orienting themselves in landscapes, communicating place-based spatial knowledge and measuring distances, improving mathematics learning and understanding through multisensory engagement. The cultural learning trajectory and PALM tool are constructivist in nature and interactive, pulling people into learn and teach by providing a stage to elicit shared local stories from the land in a way that facilitates mathematics and Tiwi learning about landscape ecology in the very diverse, cross-cultural, cross-linguistic space (Fisher, 2018b). The PALM mathematics education application is bringing digital tools and skills to life on traditional country and is building on existing meaning enabling knowledge sharing of ancient and lived experience (Fisher, 2018b).

English literacy, as well as the language of Western mathematics, the context and the way of teaching can be determining factors in the learning of mathematics for Aboriginal learners (Edmonds-Wathen, 2014). For lecturers of the GOO program it is critical to create bridges that enable the learning of mathematics for preservice teachers themselves in culturally relevant ways. The learning trajectory cognitive tool assisted by our Tiwi ‘textbook’ (PALM cultural tool) provide a culturally relevant context to bridge Western mathematics and Tiwi knowledge traditions. For this reason, this Tiwi place-based study in mathematics education will impact teaching practice on Tiwi Islands.

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