Mathematics and Coding: How Did Coding Facilitate Thinking?

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This paper reports on teachers' perceptions of students learning as part of a project examining teacher practice and student learning when using *ScratchMaths* in their classroom programmes. The project used design-based methodology, incorporating video-recorded classroom excerpts; teacher interviews; and teacher analysis and review of their practice. The teachers identified the students' problem solving, using unplugged activities, and collaborating using explicit mathematical and coding language as ways to facilitate mathematical thinking. They also recognised that their practice became more faciliatory, with their understanding of coding developing through learning with their students.

In 2017, the technology education curriculum in New Zealand (NZ) was altered to include coding as part of computational thinking, with the overall aim to develop core programming concepts. The new Digital Technology Curriculum (DTC) became mandatory for NZ primary-school aged children in 2020 as directed by the Ministry of Education ([MoE], 2017). As well as digital coding, it involves unplugged activities, using authentic contexts to develop precise, step-by-step instructions for non-digital activity, and debugging of errors that emerged as the instructions are enacted (MoE, 2017). However, research indicates that many NZ teachers would find implementing DTC challenging due to proficiencies such as coding (Crow et al., 2019; Education Review Office, 2019). Crow et al. (2019) indicated a gap in the availability of resources that are specifically situated in curriculum contexts, such as mathematics, that would practically assist engagement with coding. They also advocated that teachers and schools develop unique implementations, suitable for their school context.

This paper reports on aspects related to teachers' perceptions of student learning in a small two-year research project that examined teacher practice with coding through the use, evaluation and adaption of UCL's *ScratchMaths* resources, and the associated student learning. This two-year project aimed to support teacher learning in DTC through using, evaluating and modifying the University College London (UCL) *ScratchMaths* project resources to enhance teachers' computational thinking-based pedagogies. It also aims to address the limited resources available for teaching coding in NZ by evaluating and adapting the UCL *ScratchMaths* resources. The intention was that the project will impact positively on learners' computational and mathematical thinking. *ScratchMaths* is a highly effective resource to develop coding and computational thinking for primary-aged children in the UK school context, which is different to NZ. Hence, another aim was to evaluate and modify these effective UK resources to enhance their applicability in the NZ context.

While research has evaluated similar curriculum implementation at NZ high-school level (Johnson et al., 2017) and international research has examined some aspects of DTC (e.g., Falkner et al., 2014; Johnson et al., 2014) very little research has examined the implementation of DTC in the NZ primary-school context. There has been minimal research on the use of coding in NZ schools, hence the implementation of the DTC would benefit from being analysed by a collaborative partnership of teachers and researchers, as teachers consider its effectiveness when integrated into existing classroom practice.

Scratch is a free-to-use graphical programming environment that provides opportunities for creative problem-solving. It is a media-rich digital environment that utilises a building block command structure to manipulate graphic, audio, and video aspects (Peppler & Kafai, 2006). Studies have shown its potential for developing computational and mathematical thinking in an integrated way, particularly in geometry and algebraic thinking (Calder, 2018).

2022. N. Fitzallen, C. Murphy, V. Hatisaru, & N. Maher (Eds.), *Mathematical confluences and journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3–7), pp. 98-105. Launceston: MERGA.

ScratchMaths aims to integrate computing and mathematical thinking effectively. Coding, particularly with *ScratchMaths*, is identified as being influential on the development of mathematical thinking (Benton et al., 2018) and the understanding of mathematical ideas such as algorithms and the 360-degree turn (Benton et al., 2017). However, the *ScratchMaths* resources, while well-tested and effective resources, are designed to be undertaken by students individually, whereas in NZ learning is seen as a more collaborative, creative process (MoE, 2007). For instance, the development of collaborative student-led projects in *Scratch* (e.g., Calder, 2018), which might also emerge with *ScratchMaths*, would be conducive to collaborative problem solving. The paper reports on the teachers' perceptions of students learning through problem solving, including when debugging script, collaborating, and engaged in "unplugged" activities. It is primarily focussed on two classes of nine to eleven-year-old students, using *ScratchMaths* on laptops and iPads.

Problem Solving through Collaboration

Collaborative Problem Solving

While discussing collaborative problem solving, collaborative learning is first discussed, together with its potential to improve learning and understanding. Ways that collaboration supports learning when digital technologies are used, and the influence of both in facilitating problem solving are then briefly identified. The connection between collaborative problem solving and blending digital and unplugged experiences is then considered. Collaborative learning occurs when two or more students are engaged in an activity, interacting with each other and learning together (Dillenbourg, 1999). This perspective of learning in mathematics repositions learning more as participation in a social practice than as an acquisitional process (e.g., Sfard, 1998). Collaboration when problem solving in mathematics has been linked to enhanced understanding. For example, Mercer and Sams (2006) showed how students collaborating while engaged in tasks indicated enhanced learning outcomes in mathematics. Other studies contend that the collaborative use of digital technologies supports students in developing more flexible approaches to problem solving (e.g., Mercier & Higgins, 2013).

Mercer and Littleton's (2007) definition of collaborative learning goes beyond the sharing of ideas and task coordination to "reciprocity, mutuality and the continual (re)negotiation of meaning" (p. 23). Collaborative learning in line with this definition involves the utilization of individual understandings and expertise, with the collaborative interaction influencing the thinking of at least one participant in the interaction, even if there is only a minor adaption, coupled with a repositioning of the learners' perspective and understanding. When students work collaboratively on a task there is frequently a coordinated approach to the sense making and the approach taken when engaging with the task. The joint coordination of a task enables students to communicate and negotiate in order to support decision-making (Zurita & Nussbaum, 2004), and, as such, they are involved in "a coordinated joint commitment to a shared goal" (Mercer & Littleton, 2007, p. 23).

In general, digital technologies enable opportunities to explore and organise data to see patterns and trends more quickly in mathematical situations that might otherwise be too complex. With coding, they offer potential to learn through the interaction between the coding and the output that the coding generates. The coder can try something and instantaneously identify the effects of the new coding, enabling them to generalize coding attributes and refine their approach. With a visual environment such as *Scratch*, where the coding and output screen sit side by side, these relationships are easily identified (Calder, 2018).

Computational Thinking

Computational thinking can be considered a collection of problem-solving skills that relate to principles of computer science (Curzon et al., 2009). Abstraction, allied with logical thinking, innovation, and creativity, is considered central to the constitution of computational thinking (Wing, 2006). These elements also resonate with mathematical thinking and problem solving in mathematics. *ScratchMaths* is an interactive, intuitive space for problem solving.

Research has indicated that students become more engaged when using digital technologies, with enhanced mathematical learning also evident (e.g., Attard, 2018; Bray & Tangney, 2015; Pierce & Ball, 2009). With regards to using mobile technologies in the process of learning mathematics, Attard (2018) concluded that they improve student engagement at operative, cognitive, and affective levels. Additionally, studies have indicated that *Scratch* was an effective medium for encouraging communication and collaboration (e.g., Calder, 2018). This paper considers teachers' observations and perspectives of the students' problem solving, thinking and collaboration as they undertook coding tasks using *ScratchMaths*.

Research Methodology and Design

This research project used a design-based methodology, to examine teachers and their students' use of the *ScratchMaths* resources. The teachers were co-researchers in the process. This methodology, designed by and for educators, endeavours to enhance the implementation of educational research into improved classroom practice (e.g., Anderson & Shattuck, 2012). It can illuminate the challenges of implementation, the processes involved, and the associated pedagogical and administrative elements (Anderson & Shattuck, 2012). Design research involves multiple cycles, which include a number of different design and research activities. Nieveen and Folmer (2013) divide these activities into three distinct phases: the preliminary research phase; the prototyping or development phase; and the summative evaluation phase. These three phases, involving the teachers and including videoing of their classes, were implemented through iterations of use, reflection and modification of the resources and the associated pedagogy.

In the first year, the project involved two teachers in one primary school, who were co-teaching in a shared learning setting with 52 students. In the second year, the project included six teachers in four schools, including three new schools. This second year encompassed teachers and classes covering the junior, middle and senior levels of the primary school system (aged 7 to 12 years old). Design-based research principles and findings reflect the context and associated conditions in which they take place (Anderson & Shattuck, 2012), so providing space in the second year for new participants, in new conditions, recognised the importance of new contexts and participants emerging through the design process. Most of the data presented in this paper were related to the two teachers involved in both years.

As well as the purposive sample spanning the range of primary-school levels, there was also variance in the nature and organisation of the schools. Included in the participant schools are single classroom situations, innovative learning environments (double and larger), and a range of student and teacher-centred learning environments. The teachers also had varying levels of teaching experience and expertise with digital technology. All the schools comprised of a range of ethnicities. The research design was also aligned with teacher and researcher co-inquiry whereby the university researchers and practicing teachers work as co-researchers and co-learners (Hennessy, 2014). Allied to this was an emphasis on collaborative knowledge building. The research design was based on a transformational partnership arrangement that aims to generate new professional knowledge for both academic researchers and teachers (Groundwater-Smith et al., 2013).

The *ScratchMaths* resources identified by the teachers to use initially, were from module one and included: moving, turning and stamping a sprite, and creating circular rose patterns. The *ScratchMaths* resources and existing projects were used as starting points for the lessons, with the unplugged activities also incorporated into the sessions. Some of these class sessions and individual groups working on the tasks were video recorded. There were two iterations of the review and design process with videoing of classes each time, followed by co-researcher meetings to examine the classroom practice. One element of these meetings was the analysis of classroom video recordings. Discussions in the meetings were recorded, as were the teacher interviews. Analysis of the qualitative data from the interviews and observations was through thematic analysis with the research team identifying the initial themes, and the nodes for the NVivo analysis drawn from these themes. The data from the interviews and observations went through six phases of thematic analysis adapted from Braun and Clarke (2006).

The research question related to this paper is:

> *In what ways might the use of coding embedded within a mathematics curriculum context, influence teacher practice and children's coding and mathematics thinking?*

This research question was addressed through teachers and their students engaging with the *ScratchMaths* and modified resources, followed by teacher reflection, and re-modification of the materials and pedagogical approaches by the full research team. Episodes from the video-recorded observational data were analysed by the teacher researchers and the resources reviewed through this and their in-class experiences. Several new materials and pedagogical approaches were developed through this process.

The research project gained approval from the University of Waikato ethics committee. This approval included having all participants being invited to participate, giving informed consent (and participant assent for the student participants), confidentiality (e.g., transcriber confidentiality agreements), anonymity (e.g., use of pseudonyms), mitigation of the potential influence of power differentials, and participants' right to withdraw. Validity was enhanced through: the design of the project matching the purpose of the research questions; using a range of methods to generate the data; the design of the analysis plan; the range of contexts and participants (given the place of context in design-based research); the frequency of design iterations, the collaborative teacher/researcher research team; and ongoing peer-review of the formative findings through the research team and their colleagues.

Results and Discussion

This paper reports on teachers' perceptions of how using *ScratchMaths* facilitated the learning process in three key areas: problem solving when debugging script, working collaboratively, and using unplugged activities.

Debugging Script

The teachers consistently commented on how using *ScratchMaths* fostered a problem-solving approach as the students collaboratively debugged their codes to make them work. The process of debugging code was a particular aspect that many students became immersed in. This is a part of computational thinking that involves reviewing the code through trialling and when it didn't produce the desired output, problem-solving for possible solutions. It might also involve the output unexpectedly stopping or going into continuous loops. While the aspect of debugging was highlighted by the teachers, usually students were self-motivated with this process through wanting the script to be consistent with their expectations of the output. Marama commented on the student debugging process:

> There would not be many things that would have them that focused on what they're doing so intensely. They would be doing debugging the whole time.

The students found solutions to unfamiliar problems in mathematical contexts, through a variety of approaches. For example:

> Annie: The children were problem solving, risk taking and learning from failure

> Marama: For some activities there are no instructions for how to get them from there to there, they just had to work it out.

The students use of *ScratchMaths* within the problem-solving process at times led to enhanced engagement. The teachers identified that the students not only appeared more cognitively engaged but that the process facilitated enjoyment.

> Marama: Even though its heavy-duty problem solving, they're having fun, they're smiling and enjoying working with each other too. It's talking about what they are doing and its excited talk.

Some of the problem solving involved particular mathematical thinking. The teachers also indicated that the mathematical thinking related to both concepts and processes arose more naturally within the *ScratchMaths* activities. For instance:

> Annie: I think because maybe the opportunities with this program and what it's actually focused on with the angles and the measurement side and the negative numbers … that's probably been more cemented than what it could have been if we had been teaching it in isolation.

While the teachers made the mathematical thinking explicit to the students by referring directly to the mathematics and using mathematical language, some of the mathematics thinking emerged through attempting to solve and accomplish the tasks, and the collaboration on the coding aspects. In this way, some of the learning was initiated when the need arose, and not confined to the expected curriculum level for that age group.

> Annie: It was just-in-time learning around the maths concepts. The use of angles was very in-depth. They used negative numbers, degree turns and always mathematical language.

Negative numbers are not part of the curriculum for this particular age group. In a later discussion the teachers identified some of the other mathematical thinking that occurred: identifying patterns and relationships, exploring variations, precision with language, methodical thinking, and strategies for problem solving. Their spatial awareness, understanding of angles, and positioning sense through the use of coordinates, were all engaged to varying degrees. There was also evidence of relational thinking as students made links between their input, the actions that occurred on screen, and the effect of specific variations of size in coding procedures. They discussed how the students negotiated meaning, came to conclusions and gave explanations of what they had done. They predominantly did this through collaboration.

Collaboration

The students interacted with each other as they investigated the problems and explored potential solutions. They then collaborated on the debugging to make their codes more efficient. As they worked to design the scripts and subsequently make the codes more economical, they shared ideas and potential solutions using language that included coding terminology or was related to the mathematical or coding processes that they were discussing. The teachers noted this in the interviews. For instance, Annie indicated how the collaboration fostered their shared understanding of language, and hence from her perspective, their mathematical and computational thinking:

> Annie: So, it gives a context for social interaction to happen where they're learning to code and learning maths.

> Annie: So, then we can look at different ways of how children create a script to get to an end product and look at just simplifying the script.

Marama identified instances when students found efficient ways to code that were valued by other students, enhancing their mana (respect) within the class. Sometimes this was the students who were not usually perceived as being more capable in mathematics, so it readjusted those perceptions.

> Marama: There are kids that are capable but then someone quietly just comes up with this really simple code to do something that someone else has taken a long time to do and they think they're good so it's kind of just levelled everyone out.

This also indicated how using *ScratchMaths* facilitated collaboration. Collaborative learning can be perceived as going beyond the sharing of ideas and task coordination to the ongoing negotiation of perspectives and meanings (Mercer & Littleton, 2007). Collaborative learning in line with this definition was identified:

> Annie: It supported students' learning through communicating with friends, problem solving, increasing their mathematical knowledge and mathematical and coding language, bringing that all into the norm of how we can talk about coding.

> Marama: They're all in different roles all the time, sometimes they're teachers, sometimes they're learners.

While the ongoing negotiation and evolving perspectives are evidenced here, this also indicates that the students' roles were flexible and contingent on personal, and group understandings. Observational data also suggested that there was contestation of ideas during the collaborative work. Not only did the students interact through the ongoing dialogue as they problem solved to find solutions, but students also did at times became leaders of learning.

> Marama: One of the girls solved this thing that really no-one else was managing to do and she managed to crack it. Well, the whole class was whoosh over there, so that's fantastic that she's having to explain it and off they go all excited.

Within this collaboration, students often drew insights from the unplugged activities.

Unplugged Activities

The unplugged activities were valuable in terms of developing instructions or codes that designated actions, including movements. Some of these were repetitions, such as a series of dance moves, and some were a single task. The children wrote code that another student would enact. Once they began to trial their script it often became clearer where the debugging was needed. A teacher from the second year of the project commented.

> Katarina: When we introduced the repeat, I tried to do something unplugged with it so we did the dance. They created the five-step dance on the grid and then they had to repeat it, and they had to work out how many times they needed to repeat it for one person to complete the grid or for two people etc. And so, it's quite good for making sure that the instructions were accurate, so that everybody got the right steps at the same time to do it and to get there (the end of the grid). And then, if it didn't work ask why it didn't work, and then that introduced the debugging.

Another teacher indicated that the unplugged activities consolidated the coding moves needed and hence assisted with the coding process.

> Annie: We wouldn't have thought to use the unplugged if it wasn't in the resource but what we have found is that using the unplugged really helps to consolidate and cement in the children's minds how to create an object or whatever it is they've been asked to create. It's like the first step and then they can go and create that object on a device.

Annie also indicated the value of the unplugged activities, as students oscillated between the coding in the app and a physical activity. This helped them with developing the code and with debugging it. She identified that the unplugged activity moved their thinking.

> Annie: Also, in the teaching (of coding) when children are struggling, it's good to go back to that process so they physically do it using the unplugged... I think what I'd do more is using unplugged more.

A fourth aspect reported here is the teachers' pedagogical approach, which varied from their usual approach when teaching mathematics.

> Marama: I don't know that I need to know everything. Most of the time it's the kids that are the ones that solve things. They are learning off each other a lot more, they're going to each other a lot more, they're talking a lot more.
>
> Annie: The classroom approach is to explore, but the mathematics and coding objectives are explicit. At times (we) start with *ScratchMaths* for say, angles. There is a purposeful context for the learning.
>
> Marama: The teachers' role is facilitating learning – actively scaffolding processes and content.

The teachers were consistent in their belief that positive student learning had occurred including through student collaboration when problem solving. They were also consistent that their personal learning of coding processes had developed markedly, while acknowledging that their role in the classroom had evolved.

Conclusions

Although findings are presented separately, in practice, they were mutually influential elements that the teachers perceived had contributed to student engagement and learning. The teachers' perceptions of students' learning suggested that learning through *ScratchMaths*, including the unplugged activities, facilitated collaborative problem solving, enhancing student engagement and learning. The teachers' perceptions of their teaching approach when working with *ScratchMaths* indicated that the process influenced their practice, moving them towards a more faciliatory approach. The students' thinking and learning in coding were tied to their solving of both mathematical and coding problems, while the explicit language of both seems to have contributed to the communication of processes, concepts and solutions. These processes appeared to facilitate thinking. Students at times became leaders of the learning as well.

There was also conceptual understanding and thinking related to the Geometry and Measurement strand of the NZ curriculum; in particular, angles and spatial perception. However, the process the participants undertook, in the perceptions of the teachers, facilitated thinking through the collaborative problem-solving it evoked, and the development of logic and reasoning as they debugged code, negotiated understanding, and responded to various forms of feedback.

While the findings were limited by the size of the project, and the particular contexts in which they were enacted, they nevertheless give insights into the ways learning in coding, including unplugged activities, might be enhanced through the *ScratchMaths* resources. The research is ongoing, with the teachers in the project now leading this learning in their schools. Further research into the assessment and analysis of students' computational thinking could be undertaken which might give more comprehensive insights.

Acknowledgement. This research was supported by the University of Waikato Research Trust Contestable Fund.

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