Building Understanding of Algebraic Symbols with an Online Card Game

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The transition between arithmetic and algebraic thinking is challenging for students. One notable difficulty for students is understanding algebraic symbols—pronumerals. Researchers are exploring pedagogical approaches in seeking to address this issue. The current paper is contributing to this body of literature by illustrating how an online card matching game-based learning activity supports students' understanding of pronumerals.

The importance of eliciting students' algebraic thinking is widely highlighted in the literature (e.g., Kieran et al., 2016). Students without solid algebraic understanding potentially experience difficulties in later mathematics learning (Kieran et al., 2016). Fostering students' algebraic thinking at the transition stage between arithmetic and algebra (upper primary or lower secondary level) is generally considered a way of overcoming difficulties and helping to build a promising algebraic foundation (Carraher & Schliemann, 2018).

Previous research has listed some essential conceptual shifts during the transition between arithmetic and algebra, such as thinking of relations among quantities instead of calculation, and focusing on letters (pronumerals) and numbers instead of numbers only (Kieran, 2004). Kieran (2004) stresses that understanding of pronumerals is particularly important when students start learning algebra. According to Radford (2018), understanding of letters involves two things: understanding letters as indeterminate quantities, and understanding that letters can be operated upon in an analytical way. "Analytical way" means letters can be arithmetically operated as if they are known numbers (Radford, 2018). However, a body of literature has documented students' misconception of pronumerals (Kieran et al., 2016). For instance, students treat letters as representing specific objects rather than indeterminate quantities, such as "a" is for "apples" and "b" is for "bananas" (McNeil et al., 2010). Students also possibly assign numeric values to letters based on the alphabetical order (e.g., considering "a" is 1 since it is the first letter). Furthermore, some students are challenged by the coefficients of letters (e.g., students mistakenly consider "2h" is "2+h") (MacGregor & Stacey, 1997). The incomplete understanding of the coefficient system might contribute to students' difficulty around the manipulation of algebraic expressions (e.g., collecting like-terms).

A body of research on algebra education has been contributing to pedagogical approaches in building students' understanding of pronumerals, but more empirical research is still needed (McNeil et al., 2010). Therefore, this research aims to investigate how a card matching game-based learning activity supports students' understanding of pronumerals. The research reported upon here is a part of larger study about designing and applying a game-based pedagogy on early algebra.

Literature Review

Literature has shown that the development of understanding of pronumerals heavily depends on the pedagogical approach taken to introduce letters, particularly, the context involved (MacGregor & Stacey, 1997). One prevalent context in introducing pronumerals that may contribute to students' misconception is using *mnemonic* literal symbols (McNeil et al., 2010). This means that when presenting pronumerals, teachers use sentences such as "a" can refer to "apples" and "p" can represent "pears" (McNeil et al., 2010). Many textbooks also use

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mnemonic letters, but in a more precise way by highlighting the letter as representing quantities. For example, wording in textbooks often takes the form "a" is for the number of apples and "b" is for the number of bananas (McNeil et al., 2010). Literature shows that the context of mnemonic letters might mislead students' to consider that letters stand for specific objects rather than quantities (e.g., considering "a" is for apples rather than the quantity of apples) (MacGregor & Stacey, 1997; McNeil et al., 2010).

Furthermore, in addition to the context, building students' understanding of algebraic symbols requires overcoming results-oriented thinking from their prior arithmetic experience. Results-oriented thinking means considering an expression must have a calculated result (Malara & Navarra, 2018). Students with results-oriented thinking have difficulty accepting "lack of closure" (Kieran et al., 2016). This is to say, students consider the expressions with letters as not valid since they believe the calculation of these expressions are incomplete. In this sense, research has argued that it is important to develop students' relational view towards mathematical structures, meaning they should focus on the relation among terms in an expression rather than calculating results (Malara & Navarra, 2018).

Algebra education researchers are seeking effective approaches to teach pronumerals. For instance, Fujii and Stephens (2008) showed that students could invent informal non-literal symbols to express relations among quantities, and the formal literal symbol (letters) could emerge based on these informal symbols. Similarly, in Hunter (2010), students used everyday language to discuss the numeric relationship in arithmetic operations, progressively stepping toward using formal literal symbols in representing these relationships. Alternatively, McNeil et al. (2010) deliberately applied the counter-mnemonic strategy where instead of using "c" and "b", Ψ or Φ were used to represent the quantities of cake and brownies. McNeil et al. (2010) identified that this approach suspended students' thinking of letters as specific objects and the students with the counter-mnemonic approach outperformed their mnemonic approach counterparts in terms of understanding pronumerals as indeterminate quantities.

As mentioned, understanding of pronumerals could also include understanding that letters can be operated upon in an analytical way (Radford, 2018). This means students are able to carry out formal syntax of operation of letters (e.g., coefficient system, collecting like-terms). McNeil et al. (2010) showed that the mnemonic approach mentioned above could facilitate students' intuitive understanding of the coefficient system which hastens their idea of collecting like-terms. For instance, when a teacher uses the words "three apples and two bananas" as an analogy to $a + a + a + b + b = 3a + 2b$, students then understand the number in front of the letter is to indicate the quantity of this letter. In this sense, it is noted that the mnemonic approach could hinder students' conception of letters as representing indeterminate quantities, but it might intuitively trigger students' understanding of syntax of operating letters.

Currently, digital technology is increasingly applied to mathematics education, and so there is a call for more research about the affordance of digital technology on early algebra instruction (Kieran et al., 2016). In response to this call, this study seeks to investigate how an online card game-based activity supports students' development of the conception of pronumerals.

Methodology

This research aimed to explore how students learn with a designed game in an everyday classroom setting. The study employed a qualitative case study, which has the affordance to accommodate the complexities of natural classroom learning (Hamilton & Corbett-Whittier, 2012). Case study allows the researcher to have an in-depth understanding of what happens in students' learning processes by providing fine-grained details of students' learning (Hamilton & Corbett-Whittier, 2012).

According to Clarke (1997), to better understand students' learning in an everyday classroom context, data sources that reflect different perspectives should be complemented and triangulated against each other. Hence, three data sources were used for data collection and analysis in this study. The first one is the real-time activity recording of students' play including students' actions and conversations during the game. This data were transcribed to text for the researcher's interpretation of students' thinking. The second data source was a video-stimulated post-activity interview, which is a widely applied tool to probe students' thinking from their own perspective. The third data source was teachers' interview, which are used to infer students' learning from the teachers' perspective. The data reported here are from two grade seven students in Australia, and one grade five student in China, who are at the transition stage between arithmetic and algebra as per each country's curriculum, and all students were attending co-educational government schools. The game-based activity was conducted during a double lesson/period (about 80 minutes), as a part of students' everyday mathematics learning.

Game Design

At the macro level, the design of the game in this research was guided by a mathematics learning theory: Realistic Mathematics Education [RME] (Gravemeijer, 1994) and Gee's (2008) principle of game-based learning. RME adopts a constructivist approach and suggests that mathematics learning should start with the content which is experientially realistic to students. Here "experientially realistic" means something that is related to students' prior experiences. Students then progressively build formal mathematical understanding based on these experiences, with experts' facilitation (e.g., teachers, peers). The learning trajectory suggested by RME is in line with Gee's game-based learning principle, which argues that the game supports learning since it provides learners with a "bottom up" learning process, starting from an accessible starting point in which learners develop "performance before competence", and with ongoing experiences gradually step towards the formal abstract knowledge (Gee, 2008). Similarly, mathematics education, researchers have shown that compared to traditional paper format mathematical work, digital games are more likely to provide students with a "bottom up" process in which they gain concrete experience first, and move towards more complex mathematical concepts by reflecting and building upon these experiences (e.g., Jorgensen & Lowrie, 2012). Informed by RME and Gee, this game-based learning activity was designed as having different levels with increasing complexity, with the intention the beginning level would be readily accessible by all students.

At the micro level, the pedagogical approaches in algebra education suggested by the literature were considered. A range of research has shown that the expression/number sentence matching activity effectively supports students' algebraic thinking. For instance, Carpenter et al. (2003) used a number sentence matching activity to foster students' relational understanding of the equal sign. In the matching activity, students were more likely to pay attention to the structural relations in number sentences instead of doing calculation (Carpenter et al., 2003). Fujii and Stephens (2008) applied a similar activity to trigger students' relational view towards mathematical expressions and conception of pronumerals. In addition, exploring arithmetic regulatory (e.g., commutative law) is a good starting point to lead students to think algebraically (Malara & Navarra, 2018). Furthermore, a card matching games (like UNO®) are popular. In sum, this study designed a learning activity that has a card matching game context. The activity was designed to include four levels. At Level 1, students match simple arithmetic number sentences (e.g., $14 \times 5 - 2 - 1$ and $5 \times 14 - 3$). Level 2 included arithmetic operations but with larger numbers (e.g., $73 \times 29 + 3 - 3$ and 29×73). Level 3 mixed numbers with letters (e.g., "$793 \times 21 + 2a - a$", and "$21 \times 793 + 6a - 5a$"), and Level 4 included expressions with only letters (e.g., “a × b + 3c – c” and “b × a+ 5c – 3c”). A snapshot of the game is shown in Figure 1.

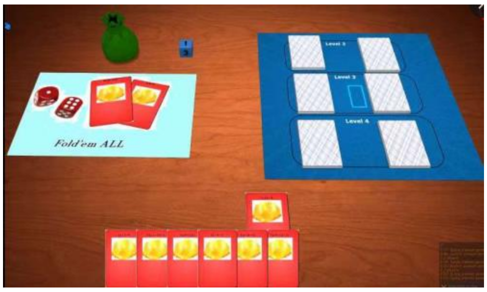

Figure 1. A game snapshot.

According to RME, Level 1 and Level 2 provide students with an “experientially realistic” starting point, since participating students should be very familiar with the arithmetic context. At Level 1, students can simply calculate answers to compare cards, but at Level 2, the calculation becomes more complicated since numbers are larger. Within this setting, it is hoped that this level could push students to start considering the structural relations between number sentences on cards. It is expected during these two levels students gain experience in focusing on considering the relation among quantities in mathematical structures rather than calculating results, so the results-oriented thinking could be suspended. As mentioned, overcoming results-oriented thinking is essential to understand the expressions with letters. At Level 3 and Level 4, the participating students, who have no prior formal algebra experience, need to compare and match expressions with letters. Here it is hoped that equipped with the experiences gained during previous levels, students are likely to possess a disposition to look at the algebraic expressions with a relational view instead of as sequences of calculations. By doing so, it is expected that even without the formal learning of expressions with letters, students, at least, could accept these “uncalculatable” mathematical structures as legitimate, and start comparing the structural similarities. Furthermore, as mentioned, an inappropriate context such as using the mnemonic literal symbols to introduce pronumerals tends to mislead students. In this sense, this game is designed to create a context in which students see letters come out with numbers together in the first instance, and they will be immersed in a context of mixing letters and numbers through engaging in the entire game. It is expected that in this pure numeric/algebraic context, students could intuitively draw a connection between letters and numbers.

Results and Discussion

Due to space restrictions, only one student’s (Fiona) data will be reported and analysed in detail. Fiona’s data illustrates the typical findings of this study. In addition, two other students’ (Michael and Hanwei) data will also be mentioned as a supplement, to demonstrate diversity in findings. Based on their teachers’ comments, all students were considered of average mathematical ability according to their level of schooling, and they had no formal algebra experience before the activity. The data reported here are episodes from Level 3 and Level 4 of the card game in which three students were exposed for the first time to expressions with letters.

Fiona

When entering Level 3, Fiona had gained extensive experience in comparing number sentences with considering relation among the numbers rather than doing calculations. For example, when comparing 29 × 73 – 3 and 73 × 29 – 2 – 1, she recognised the multiplication parts were the same without calculation but due to the commutative law, so she only calculated

"– 3" and "– 2 – 1" then accepted two number sentences as the same. When Fiona saw algebraic expressions at Level 3, she spontaneously compared cards without asking what the letters meant. For instance, Fiona matched a × (793 + a) and a × (a + 793), and her explanation is shown,

> Because they had the same start, like "a" times and the bracket. And then they got the same number in the bracket, just switched around.

In the interview, Fiona was asked whether she worried about the value of "a", and she answered that she did not worry about the value of "a", because both sides had the same "a". This episode tends to show that Fiona was able to compare them without knowing the value of "a" because she did the comparison by considering relations among terms. The words such as "same" and "switched around" further indicated Fiona focused on the structural similarities of expressions rather than calculating the results (Malara & Navarra, 2018). Fiona's explanation highlighted that she compared the expressions part by part, suggesting Fiona treated these expressions as standalone entities instead of sequences of calculations. This could be evidence of Fiona's acceptance of lack of closure. As mentioned, acceptance of lack of closure is a precursor to the understanding of pronumerals as indeterminate quantities, as it indicates students are beyond the results-oriented thinking. It could be argued that Fiona had extended her disposition to look at number sentences with a relational view developed during the previous levels to Level 3. Also, Fiona achieved this without any formal algebra instruction. This tends to suggest that Fiona's experiences gained at Level 1 and Level 2 took effect at Level 3.

However, two expressions in the episode above did not require collecting like-terms. In a later episode, Fiona found confronting when for the first time she encountered more complex expressions that collecting like-terms was needed, which were for example, 791 + 2 + 3a – a+ b and 793 + 2a + b. Fiona completed the comparison with the teacher's prompt. The excerpt is shown below,

> Teacher: How many "a" here, you have three "a" then you take away one "a", How many "a" here in total?
> Fiona: Em [pause] two
> Teacher: So how many "b" here?
> Michael [Fiona's teammate]: One
> Teacher: So which card you are looking for now?
> Students seek cards, and point to 793 + 2a + b
> Fiona: Yes, here it is.

With the teacher's prompt, Fiona, who had no previous experiences with formal algebra, could operate letters in an analytical way. When the teacher asked how many "a" were left, Fiona was able to recognise there will be two "a". In the interview, Fiona further explained why she selected 793 + 2a + b to match 791 + 2 + 3a – a + b, as shown below,

> Because at the start, it's seven nine one plus two and the seven nine three that is the same, and then plus two "a" and this is plus three "a" minus "a" that's two a, and plus "b", plus "b".

Fiona's explanation clearly illustrated her conception, which was that she was able to evaluate the expressions with letters part by part and operate the letters in an analytical manner. In this episode, Fiona's teacher Mr I played a role to facilitate students' comparison. The language used by Mr I was similar to the early mentioned mnemonic approach, which is, for example, analogising "a + a" as adding two "apples" in total. This study showed this kind of language was effective to make students understand how these letters could be operated. This kind of language constitutes the notion of "algebraic babbling", coined by Malara and Navarra (2018), which is used to describe a situation in which algebraic ideas are built upon using natural language. According to Malara and Navarra (2018), learning formal algebra can be bridged by using natural language in which underlying algebraic ideas are possibly situated. Here, when using language "three a" then you take away "one a" to describe the expression "3a – a", the teacher was trying to convey the message to students that the number in front of "a" refer to the quantity of "a". Then the formal pronumerals syntax of "3a – a" can be portrayed as "three lots of something take away one lots of something." It appeared that Fiona grasped this idea. In later episodes, Fiona was able to consistently apply this strategy to simplify the expressions to do the comparison.

It appears that when attention is paid to this kind of language, students may grasp the idea of operation upon letters, but it is not sufficient to support students in understanding letters as indeterminate quantities. A student might be able to operate collecting like-terms without understanding the meaning of pronumerals. For instance, as mentioned above, a student who considers "a" as "apple" is still able to do "a + a = 2a" (see McNeil et al., 2010). To this end, in this research, the teacher avoided to use mnemonic words such as "apple" and "banana", instead, the teacher used the letters as they were presented, not denoting a specific object as such. By doing this, this research tried not to mislead students to consider the letters as objects. However, it is noted that in the language used by the teacher, the conception of letters as indeterminate quantities did not explicitly stand out. In this sense, as mentioned, this activity was designed to let students see that letters came together with numbers as they were immersed in this context at Level 3 and Level 4 of the card matching game, so it was hoped that students would intuitively draw the connection between letters and numbers. In the interview, Fiona was asked about the meaning of the letters:

Interviewer: What do you think these letters represent for?
Fiona: Amounts.
Interview: Any mounts or only can be a fixed number?
Fiona: Any number.
Interviewer: Okay, you see the expressions have the letter "b". May I use "d" or "e" any other letters to replace "b" here?
Fiona: Yeah, I think it would be the same.
Interviewer: Why?
Fiona: Because it wouldn't matter what letter of it is. It just the first number that matters.

This except shows that Fiona perceived letters as "any number", indicating she understood the meaning of letters as representing indeterminate quantities. Furthermore, Fiona showed that she understood that it was not important which letter is to be used but the coefficient ("the first number") of the letter did matter, without being told by the teacher. In the interview, Fiona also indicated that when she was playing Level 3 and Level 4, she continuously worked with numbers and letters together, so she "felt" a letter might stand for a number as well. Also, Fiona said since she did not need to "worry about" the value of letters, she thought the letters could be any value. Fiona's teacher Mr I revealed that when his class started learning formal algebra after this activity, Fiona could easily understand the formal definition of pronumerals. In fact, many other participants (12 out of 16 participants) in this study had similar responses as Fiona, commenting that letters can be "random numbers" or "any numbers." This appears to indicate that when students are immersed in a context mixing the numbers and letters, it is possible that they intuitively draw the connection between letters and numbers, and they are likely to perceive letters as quantities. This tends to suggest that in this study, the context to introduce pronumerals, which is mixing letters with numbers, had some effect.

This study also found out that the three students were able to notice letters as quantities because they considered the letters as specific numbers such as "1". However, as MacGregor and Stacey (1997) suggested, the context to introduce pronumerals needs to be appropriate as it plays a key role in building students' understanding of pronumerals. In this study, the context to introduce the pronumerals appeared to be an "appropriate" context, in which students, at least, could appreciate letters as quantities.

Hanwei

Hanwei had a similar progress in the game as Fiona. Hanwei was able to spontaneously match the expressions with letters by comparing the structural similarity. With peer support, Hanwei was able to analytically operate letters as if they are known numbers, to simplify expressions. However, when Hanwei was required to explain the meaning of letters, he answered “these letters represent some numbers, for example, “1”, and “the letters cannot be any numbers, just for one number.” This suggests while Hanwei perceived the letters as numbers, he did not realise they stand for indeterminate numbers. Interestingly, the following excerpt revealed some further insights of Hanwei’s conception of pronumerals.

> Interviewer: If you think these letters represent specific values, why you did not substitute these values into the expressions to calculate the answer then do the matching?
>
> Hanwei: I don’t have to do this, because it does not matter what values are, it is always quicker to just add or minus the number in front of these letters if both sides have the same letters.

Hanwei’s response reflected that while he considered these letters as specific values, he still appreciated some generalities and relations of letters. He noticed that these letters could be directly operated on analytically and it did not matter what values the letters represented. Hanwei’s data showed that he appeared to have some understanding of letters whilst he could not fully appreciate the meaning of pronumerals. This is in keeping with what Malara and Navarra (2018) have shown that when students are first introduced to algebra, they might be developing a certain level of understanding, but, it could still be naïve and tentative, meaning that errors and misconceptions might still occur, so teachers’ further support is needed. Hanwei’s teacher, Ms Q revealed, that after the game, this class was learning the topic of perimeter of square and rectangle, and Hanwei easily understood the meaning of the letters in the formula $(a + b) \times 2$, as “a” could be any length and “b” could be any width of a rectangle. In this sense, this study argues that Hanwei’s partial understanding of pronumerals gained in the activity facilitated his later complete understanding. As Ms Q commented, she believed the game in this study built a foundation for Hanwei’s latter formal understanding of pronumerals. As mentioned above, Gee suggests the importance of “performance before competence” in learning, and RME argues understanding formal abstract mathematics knowledge can be built on students’ realistic prior experience. Hanwei’s case shows that the designed game has a potential to provide students with some basic sense of pronumerals which could be a base for their full understanding of letters.

Michael

Michael exhibited similar progress in the game as Fiona. However, Michael was the only student who did not recognise letters as a quantity despite being able to operate analytically with the letters. In the interview, Michael revealed that he did not know what the letters stood for, and only operated them as they were. Like the case of Hanwei, Michael’s case showed that a student can collect like-terms to simplify algebraic expressions but not necessarily understand the meaning of pronumerals. Nevertheless, in the interview, Mr I revealed that Michael’s relational sense that emerged in the activity still benefited Michael in understanding pronumerals when formally learning algebra later. Mr I’s words confirmed that the game, supported students in overcoming results-oriented thinking when developing their understanding of the pronumerals.

Conclusion and Limitations

This study contributes a pedagogical approach for understanding formal algebraic symbols (pronumerals) by arguing the designed game is supportive for students’ conception of pronumerals in several ways. First, with a “bottom up” learning trajectory suggested by RME and Gee, students were gaining experience in viewing mathematical structures in a relational way beyond merely doing the calculation at Level 1 and Level 2 which is the accessible starting points for students. Equipped with these experiences, students possessed a disposition to look at expressions with letters at Level 3 and Level 4 as relational structures rather than sequential calculations, therefore they accepted of lack of closure and compared these expressions with structural similarities. Second, the data suggests that the context mixing numbers and letters is effective in facilitating students to build an intuitive impression that letters represent quantities. Third, students' understanding of coefficient system of pronumerals could be triggered by teachers' natural language that constitutes the notion of "algebraic babbling". Finally, this study finds that some students' conception of pronumerals was still naïve or incorrect despite that they were able to analytically operate these letters. However, it appeared that the experiences gained in the game provided these students with a basic sense of pronumerals, so had foundations on which to build when formally introduced to algebraic symbols. The effectiveness of this pedagogical approach needs to be further investigated in a large-scale study, where perhaps quantitative research is desirable.

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