

## BOOK REVIEWS

### **Fractions in Realistic Mathematics Education: A Paradigm of Developmental Research.**

Leen Streefland

Dordrecht, Kluwer, 1991, ix, 354 pp.

This book presents a description of a 3-year study of the teaching of fractions at the primary level in the Netherlands which followed on from the Wiskobas project reported in Treffers (1987). A constructivist approach was followed in the teaching experiment and thirteen children were followed in detail throughout the project. Their contributions to class discussions, assignment work and test results formed the basis of the descriptive account in the book and the research conclusions reached.

The theoretical model for constructing the course used in the research project was based on three levels derived from van Hiele: the classroom level or horizontal mathematization, the course level or attaining higher levels through vertical mathematization and the theoretical level. Similarly the specific course was constructed on a three-level principle: first providing mathematical and instructional substance to the constructive handling of meaningful concrete material, second generating the relations between the fraction materials, and third developing the relations on a formal mathematical level. A chapter is devoted to describing how the fraction course operated in theory and practice. The constructivist standpoint is well illustrated in the following passage.

Fractions ... are gradually used as a means of organization in order to mathematically describe the results of breaking, fair-sharing, etc. No arithmetic recipes are offered for the fractions and their mutual connections. As we shall see, the phrasing of the questions we have in mind leads to informal actions right from the start. This type of insightful, non-regulated action is encouraged as much as possible. In this way are concept-formation and insightful actions connected inseparably - and are therefore mutually meaningful - from the very beginning. Formalization of the actions takes place gradually and is marked by *five* clusters of activities, which proceed more or less simultaneously during the learning process and which, therefore, cannot easily be distinguished from one another. (p. 48)

The five clusters are as follows:

1. Serving up and distributing (producing fractions and their operational relations).
2. Seating arrangements and distributing (intertwining with ratio and generating equivalencies).
3. Operating through a mediating quantity (the four main operations).
4. Doing one's own productions at a symbolic level.
5. On the way to rules for the operations with fractions. (p.48)

Following an extended discussion of the operation of the course with numerous illustrations of students' work, another chapter is devoted to a student by student analysis of the fraction research (by the children) and constructions which took place over the three years. For seven children with a range of abilities the descriptions were classified by five indicators of the learning process:

- concept acquisition (fraction, ratio) and IN-distractor errors (1);
- progression in schematizing (2);
- flexible use of models, application of diagrams (accompanied by flexible calculations) (3);
- ability to build mental images of formal problems (4);
- individual constructions and productions on a symbolic level (5). (p.136)

Again, examples were used to show the children's progress through the five indicators, which obviously had a close relationship to the van Hiele levels. Six other children were described more briefly at the end of the chapter.

The work of the children was then combined to extract clues as to the development which took place during the three years. This analysis basically followed the progress in relation to the five indicators noted in the previous paragraph but also with reference to the three developmental levels mentioned at the outset. In some cases conclusions with respect to the levels could be represented graphically for the 13 children and in other cases it was only possible to describe and illustrate performances associated with the levels. Several specific conclusions are noted in the midst of this developmental approach.

With reference to "progression in schematizing (2)" it is concluded "that schematic solution of comparison problems is not at all easy for students of this age" (p.232) and it is claimed that the "ability to build mental images of formal problems" indicator makes it "possible to check that students are not yet ready for the formal-symbolic third level or, put another way, are not yet able to rise above the - for many - laboriously achieved intermediate level" (p.240). One rather surprising conclusion condemned two of the children never to succeed in forming rules for operations and formal treatment (p.239). On the whole, the 13 students in the study were not able during the three years to reach the third level of formal operations with fractions but in general it was not presupposed that they would not do so at a later stage.

The descriptions of the course and the acknowledgement of possible instructional deficiencies are refreshing, as often researchers are very defensive about what they have set up to be tested. After reaching the conclusions of his study Streefland goes on in an epilogue to suggest a 'new course' for fractions growing out of what was learned in the three year study. The outline, covering more topics than taught in the research project would appear to offer promise and could form the basis for research by others who are interested in the study of fractions (up to and including decimals).

The external evaluation of the course took place via a comparison of the 13 children in the research group with other classes of students who had been taught over the same period using various well-known Dutch text book series, categorized either as realistic or mechanistic. These were described in considerable detail as was the test which was devised to compare the students. The test consisted of 10 items, only the last of which monitored traditional operations with fractions. The others gave opportunity for children to demonstrate their ability to devise models, use symbolism and use clever or direct calculations. Each item was analyzed and except on the last the experimental group outperformed the other groups. Although perhaps not a surprising result, it is discussed in the light of the expectations of the courses and comment is made on the other two types of instruction, common in Dutch schools. Readers in other countries will undoubtedly feel that similar results would occur in their countries.

The final chapter is a theoretical review of the implications for the teaching of fractions, tying together educational principles with the clusters of activities involved in the instructional process and the indicators of the learning process. This is done twice: for the concrete and higher levels. While at the beginning of the book (pp.46-7) van Hiele's view that bare fraction problems should be abolished in primary school is questioned because he did not consider how the concrete level could be accommodated in a sound manner, by the final chapter it is recommended "that no level higher than the second one should be strived for in primary school fraction education" (p.319). Dropping this formal goal, attention is then turned to progressive mathematization which offers students the opportunity to reach whichever levels they are capable of attaining. This leads to the suggestions for an improved course which completes the developmental or action research cycle.

Although the translation into English was generally adequate, there were some unfamiliar phrases and inconsistencies in the type setting and highlighting of points which made reading less straightforward than it might have been. Despite the implicit use of van Hiele levels throughout the book, these are not specifically outlined as a model for use with the instruction and learning of fractions. This assumption of prior knowledge on the part of the reader also relates to the discussion of IN-distractors, which have an important part to play in the first indicator of student learning. It is possible to pick up from the context that IN-distractors are those ideas which lead children to apply whole number rules incorrectly to fraction problems to

obtain results like  $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$ . A book of this calibre, however, should be self-contained in terminology, particularly since many of the author's referenced works were not written in English.

Despite these few drawbacks there is much to be gained from a careful reading of the book, especially for curriculum planners and text book writers. Constructivists will be pleased to see their approach to instruction and learning taken so seriously and with such generally positive results. It is likely that developmental psychologists will not be quite as satisfied with the results of the study. This reader was left with a feeling that the voluminous data analyzed have not quite been organized to clearly show the forest rather than the trees.

Those who have argued in recent years that fractions have a diminishing role to play in the mathematics curriculum will be interested to read Streefland's attempt to frame the topic in a realistic fashion, as is claimed in the title of the book. Although he has gone a long way in this direction, there will still be those who remain unconvinced that the context is realistic enough to warrant the continued devotion of so much of the mathematics curriculum to the topic.

Whoever wrote the piece on the back cover of the book got it wrong when s/he said "Indeed, the book does not deal with fractions at all despite its title." Indeed, this book does deal with fractions from cover to cover - the topic is obviously close to the heart of Leen Streefland. He has seen their value for learning the mathematizing of relations if built upon the proper concrete foundations and he has put their study within the developmental research framework; but the book first and foremost is a book about fractions. All who would carry out research in the area will gain from the effort put into understanding Streefland's work.

#### Reference

Treffers, A. (1987). *Three dimensions: A model of goal and theory description in mathematics instruction - The Wiskobas Project*. Dordrecht: Kluwer.

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