Developing Guidelines for Teachers Helping Students Experiencing Difficulty in Learning Mathematics

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As part of an ongoing project, we have developed a model of planning and teaching that is designed to assist teachers to help students overcome barriers they might experience in learning mathematics. The following is a discussion of one aspect of the model that we term “enabling prompts”. These refer to the directions, invitations, or questions that a teacher offers when interacting one-on-one with students experiencing difficulties. We argue that teachers should plan to pose subsidiary questions in the first instance, rather than, for example, offering further explanations. We outline our overall planning and teaching model, we present some examples of enabling prompts used by our project teachers, and we propose some considerations for teachers when structuring their own enabling prompts.

Student Difficulties as a Challenge for Teachers

The first author presented a group of secondary school teachers with a set of seventeen indicative steps that were intended to represent teacher actions during the genesis of a lesson. The teachers were given the option of adding, deleting, or changing the steps proposed. There were very few such changes suggested, indicating that the teachers accepted the generic steps as illustrative of typical lessons. The teachers were asked to rank the three most important steps. Two of the seventeen steps were rated much more highly than the others. These were:

Step 1. Team meeting to develop overall plan and to consider the goals.

Step 12. Teacher moves around interacting with individuals assisting their progress, identifying difficulties or misconceptions, extending those who are ready.

Our sense is that the first of these steps is adequately and appropriately addressed in initial teacher education programs. We are not so sure about the second.

The teachers were also asked to rank the three most important from a set of 11 “thoughts that a hypothetical teacher might have when about to interact with a student one on one”. Far and away the most commonly selected “thought” was “This student can learn”. The next most common thought was “If a student is experiencing difficulty I will not tell them what to do—instead I will ask another question”.

The high ranking of this latter statement reflects two phenomena that are prominent in our current work. The first is that one-on-one interactions between teachers and pupils, even when those interactions occur in larger groups, seem to be more personally significant for students than general interactions with the class as a whole. Second, there is considerable ambiguity in advice to teachers about the nature of their own contribution to interactions with students, especially if they are experiencing difficulty.

We suspect that neither the relative importance nor the nature of interactions with individual students is given due emphasis in teacher education programs, that the roles of teachers in these respects are under theorized, and that neither is given prominence in advice to teachers. For example, no doubt readers have attended presentations to groups of teachers when teaching ideas or lessons are outlined. Seldom are suggestions made on how to assist students experiencing difficulty with the idea presented. Presumably it is assumed that teachers can work out for themselves how to help such students. Our current research suggests that this assumption is unwarranted, and there is a need for more explicit attention to both the theory and practice of assisting students experiencing difficulties.

Current Advice

The limited advice that does exist on this aspect of teaching is contestable. The advice ranges from providing explicit re-teaching to individuals, to working intensively with small groups, to grouping student experiencing difficulty together. Ellis (2005), in a review of the psychological literature of teaching students with learning difficulties, argued that students experiencing difficulty should be given direct instruction. Ellis emphasised explanations, including scripted presentations, teaching essentials, and small group instruction, and she recommended rapid pacing and drill. She argued that direct instruction is significantly more effective for mathematics teaching that what she termed constructivist instruction. However, we see more direct instruction of struggling students as potentially counter productive, because if students have not learnt from the explanation(s) that have been given already then further explanations are unlikely to help. Further, if the rest of the class is learning by working on tasks that allow them to create knowledge for themselves, there is no reason why students experiencing difficulty should not learn in this way, so long as the task they are working on is appropriate to their level of development.

The Victorian Department of Education, Employment and Training (2001) proposed that teachers work intensively with small groups to deal with the needs of individuals. We see this as potentially disadvantaging students experiencing difficulties since they are separated from the rest of the class, and it is obvious to them that this is the case. The same argument as above also applies: if other students are learning from constructive activity, it does not seem logical that students experiencing difficulty would learn better by listening to explanations. Further, the bulk of the students usually subsequently engage in a discussion of their explorations and if the students experiencing difficulty have not undertaken their own explorations they would be excluded from that part of the lesson as well—or at least be unlikely to engage personally and contribute productively.

Neither do we consider the practice of grouping students by teachers' perception of their ability to be the answer. A commonly observed effect of this is reduced opportunities for students in the lower groups (Zevenbergen, 2003). This can be due to self-fulfilling prophecy effects (e.g., Brophy, 1983) and effects of teacher self-efficacy, which is the extent to which teachers believe they have the capacity to influence student performance in particular groups (see Tschannen-Moran, Hoy, & Hoy, 1998). Brophy (1983) argued that rather than grouping students by their achievement teachers could: concentrate on teaching the content to whole class groups; keep expectations for individuals current by monitoring progress carefully; let progress rates rather than limits adopted in advance determine how far the class can go; prepare to give additional assistance where it is necessary; and challenge and stimulate students rather than protecting them from failure or embarrassment. Dweck (2000) outlined a detailed theoretical justification for this advice.

We argue that students are more likely to feel fully part of the class if teachers offer appropriately structured prompts to allow those experiencing difficulty to engage in active experiences related to the initial task, rather than requiring such students to listen to additional explanations or assuming that they will pursue goals substantially different from those of the rest of the class. Adapting carefully selected core tasks to provide appropriate problem solving opportunities to students experiencing difficulty is evident also in the recommendations of Gervasoni (2003), Griffin and Case (1997) and Ginsburg (1997).

The "Overcoming Barriers" Project

The discussion below arises from research seeking to identify strategies to engage students in learning mathematics, with a particular focus on students disadvantaged due to socio-economic factors or missed prior learning opportunities.

Initially we identified and described aspects of classroom teaching that may act as barriers to mathematics learning for some students. Next, we described strategies for overcoming such barriers (see Sullivan, Zevenbergen, & Mousley, 2002), including creating some partially scripted experiences taught by participating teachers and analysed by us (see Sullivan, Mousley, & Zevenbergen, 2004). This analysis allowed reconsideration of the emphasis and priority of respective teaching elements. We found that it was possible to create sets of experiences that could be taught as intended by teachers, and that many of these experiences had the effect of including most students in rich, challenging, and inclusive mathematical learning.

Arising from this work, we developed a model for planning and teaching mathematics. The key elements of the model are as follows:

The tasks and their sequence

Open-ended tasks create opportunities for personal constructive activity by students, and appropriate of sequencing of tasks contributes to their effectiveness.

Enabling prompts to support students experiencing difficulty

Teachers offer *enabling prompts* to allow those experiencing difficulty to engage in active experiences related to the initial goal task, rather than requiring them to listen to additional explanations or having them pursue substantially different goals.

Extending prompts for students who complete the initial task readily

Students who complete the planned tasks quickly are posed supplementary tasks that extend their thinking on that task.

Explicit pedagogies

Teachers make explicit the usual practices, organisational routines, and modes of communication that impact on approaches to learning, types of responses valued, views about legitimacy of knowledge produced, and responsibilities of individual learners.

Learning community

All students progress through learning experiences in ways that allow them to feel part of the class community and contribute to it, including being able to participate in reviews and class discussions about the work.

Details of how each of these elements operates are contained in Sullivan, Zevenbergen, and Mousley (in press). This article seeks to elaborate the issue of *enabling prompts* as a way of assisting students experiencing difficulty.

Examples of Enabling Prompts

To illustrate what we mean by *enabling prompts*, assume that a teacher had planned to pose the following task for a class:

> A school arranged 6 buses to take all students and teachers to the swimming sports. Someone noticed that when there was the same number of students and teachers on each of the 6 buses there

were 4 students left over. How many students and teachers might there be in the school?

The intention with this task is that the students will explore aspects of multiples of 6, as well as considering the meaning of a remainder, and that if these issues are not apparent to the students from their work on the task then the subsequent class discussion can provide further learning opportunities. Like many good open-ended tasks, it presents an opportunity for students to recognise and use algebraic generalisations.

It is assumed, for the purposes of this discussion, that the teacher does not plan to explain to the students how they should solve the task although s/he will ensure that the context and any difficult terms are clarified. The teacher might anticipate that some students could experience difficulty with the task as posed. Some steps that might be taken in advance could be to:

- plan to pose a simplified version of the problem, such as:

  A school arranged 6 buses to take all students and teachers to the swimming sports. Someone noticed that when there was the same number of students and teachers on each of the 6 buses. How many students and teachers might there be in the school? (Note that this could be simplified and scaffolded even further if necessary; e.g., 2 buses or 3 buses.)

- have a work sheet with 6 buses drawn, and plan to ask students experiencing difficulty to write in (or draw if necessary) numbers people to show what the buses might look like; and/or
- have some containers of counters and 6 pieces of card, and plan to give them to students experiencing difficulty and ask them to imagine that the counters are people, and the pieces of card are the buses, and invite them to model the situation.

Even though such alternate enabling prompts have been prepared, the teacher still needs to make a judgment on what enabling task to pose first to a particular student experiencing difficulty. The question here is what the child might benefit from; i.e., which element should be reduced by one level of complexity. Our research has shown that relevant elements, here, include language and reading or listening skills (e.g. reading or comprehending the task, semantic features, or word elements such as “left over”); mathematics concepts (e.g. same number); contextual knowledge (such as buses not holding 100 people); mathematics skills (where reduction of numbers will help); the ability to cope with complexity (where starting with the one-stage problem will help); and difficulty in connecting the words of a problem to a practical situation (where concrete materials or drawings will help). There are also psychomotor considerations (e.g., aids may be too difficulty to manipulate). Thus a decision needs to be made when planning lessons about specific types of barriers that one of more students may experience, and during lessons about difficulties being experienced.

We are not arguing that further explanations or questions are not of any use, but that they need to be targeted so that they focus on the actual barrier being experienced by the child. We have found that, when this happens, questions or short and focused explanations become enabling prompts, as do invitations to act such as those described above.

Essentially our model invites teachers to consider alternate prompts that they can pose to students experiencing difficulties—prompts that will enable them to work on their own to overcome specific barriers. Note that we do not recommend that the teacher start the lesson with these sub tasks, intending to move to the main task eventually. This would only have the effect of retarding the bulk of the class who we assume are ready for the task as posed, and unnecessarily removing opportunities for developing problem solving skills and intuitive performance. It is also important not to reduce the task more than one conceptual step at a time because the aim is to keep students thinking at their optimal level while they overcome their barriers, enabling them to tackle the original tasks and hence remain a part of the learning community.

Some Descriptions from Lessons Including Enabling Prompts

We have gathered broad ranging data over different phases of the project on all aspects of the planning and teaching model. Initially we provided considerable direction to teachers, and more recently teachers have used the model to create their own lesson sequences. We collected data though teacher self-report, through structured observations, and through analysis of student work.

As it happens, these forms of data collection are holistic and address multiple elements so are not conducive to capturing the subtleties of one-on-one interactions between teachers and pupils. We can see teacher interactions and we can see that the interactions are less likely to be instructions that they were initially, but we have not been able to capture adequately the forms of difficulties that students experience, and the specific types of prompts teachers use to assist. It is worth adding that we would not expect *enabling prompts* necessarily to occur in every lesson, especially if lessons are based on open-ended questions that students can access at their own level and where being open-ended includes open entry as well as possibilities for extension and generalisation.

We have been able to get some insights into the ways teachers respond to individual students through teacher self report. For example, the following is a description given by one of the project teachers:

> But it was interesting, one kid chose the hexagonal vase because he liked the way it looked. But then when it came down to working out the volume he had no idea on how to calculate the area of the hexagon and I suggested he could break it up into some other shapes: "What shapes could you break it up into?" No idea. He talked about what area was and we talked about area in terms of being centimetres squared or millimetres squared, so he traced around the shapes and he decided to break it up into half centimetre squares, because centimetres squared would be too big for the shape.

While ideally we might prefer this to be slightly less directive, the teacher was still considering the progress of an individual student, she posed an alternate task and then a more concrete one, and the student was able to progress with the task of finding the volume of an interesting container.

We also sought data from direct observation of classes. The following is a report from a trained observer of the class of one of our project teachers, focussing on the topic of subtraction.

> Once the class was set to work, Peter (pseudonym) then engaged individually with the students:
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> Peter positively acknowledged students' queries and attempts: "Nine, well done!" [referring to the numbers of responses] "Yes, well done, there could be ten …" "Good question, does the answer need to be a whole number? … No it doesn't have to be …" "Are you going to leave it as a decimal or a fraction?"
>
> He continued to assist around the room: "Kerry, you're looking puzzled. What could you put there? … Minus one, yes. What might the answer be? Nine…. Peter noted John's "lovely system." In reply to another student's query, Peter suggested that a system would make the task "nice and easy to follow".
>
> Peter kept assisting students around the room. "Alec, use my calculator. Does anyone else need a calculator?"
>
> Students asked Peter how many examples they needed to do. "How many?" Peter replied

> humorously, "For you fifteen, everyone else three!" Students were quietly engaged in this activity while Peter coached students as needed.
>
> One student said, "I don't like carrying figures!" Peter: "Sorry buddy, if you haven't got them in, I'll say you've cheated."
>
> Peter re-focussed a boy at the front table by coaching him, using a calculator, and reminding him to do maths first before resuming his drawing activity.

Such responses directed students' attention to elements of the task and helped to maintain their engagement, as well as proposing variations that could assist those experiencing difficulty. The task itself was graduated and so specific task variations were not necessary, but some of his comments did suggest a challenge.

The following data were taken from two lessons where the main prompt was not a question, but an action, which of course is easier to observe.

The first lesson is described more fully in Sullivan et al. (in press), and was about representing 3 dimensional shapes in 2 dimensions using isometric grid paper. As an introductory task, the students were asked to use the paper to draw shapes that could be made using 3 cubes each with at least one face touching another.

There were two suggestions in the teachers' notes of enabling prompts that were ready to be offered to students experiencing difficulty. The first prompt was to "Have some isometric paper with the 2 (cubes) already drawn". In Sullivan et al. (in press), this was described as follows:

> … there were students who had not appreciated the way the isometric paper could be used. Interestingly, some of the students experiencing this difficulty may have been good at drawing in perspective, and some used the dots to draw a conventional perspective drawing. They were aware, however, that their drawing was not working as intended. This created anxiety for a number of reasons not the least of which was that they did not like having messy work. Once they became aware that their drawing was not what was intended, they sought to erase their work. Some additional isometric sheets with two cubes already drawn had been prepared. The teacher merely replaced the sheets of the students experiencing this difficulty with this new sheet. In all cases, this action was all that was needed.
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> It seems that the combination of the fresh start, and the hint provided by the cubes already drawn allowed all students to complete the task. … The difficulties of these particular students appeared due to uncertainty about the form or the conventions for the representation. …In retrospect, the ultimate apparent inclusiveness of the lesson overall and the progress of all students onto the ultimate goal task is perhaps due to the effectiveness of this prompt. It is worth noting that using the prompt in this way is likely to be more productive than merely having all sheets with one shape drawn already, in that it allows those students who are capable to work the drawing convention out for themselves. The students who tried but did not complete the drawing then had their attention more directly drawn to the requirements of the task.

The second enabling prompt was to "Have some cubes. Ask them to model what the 3 (cubes) might look like". Sullivan et al. (in press) described the effect of this prompt as follows:

> (It was) suggested to students who had drawn one configuration appropriately but seemed to be experiencing difficulty in visualizing what another would look like. This difficulty may have been because the task was artificial … or because the challenge of visualizing of an alternate configuration was great. The teacher used three large cubes to model the less obvious configuration on the teacher's desk: this was drawn to the attention of these students. No further prompt was necessary, and all were able to proceed with exploring the mathematical task.

Both of these prompts proved to be ways of supporting the learning of individual students experiencing difficulties on a one-to-one basis, without resorting to extended explanations, without in any way emphasising the difficulties that some students had in commencing the task, but allowing them all to experience the same success as other students.

Another lesson had a goal of asking students to draw a variety of triangles with an area of 6 sq cm on squared paper. The first step was to invite students to draw letters of the alphabet with an area of 10 square units. As an enabling prompt, a sheet with a letter L covering 10 squares had been prepared. The observer's record of its use was reported in Sullivan et al. (2004) as follows:

> Teacher A had prepared some sheets of squared paper with one letter of 10 square units already drawn. In the observed lesson, the task was introduced but only a few students were able to commence so Teacher A distributed the additional sheet to the others. Having a model to follow was enough to allow them to commence. Teacher A had also prepared some squares cut from card, to allow students to drop back to a more basic entry point, but these were not needed.

These are merely illustrative reports of teacher one-to-one actions that we have observed on many occasions from our project teachers. We have concluded that not only is it possible, within time and other classroom constraints to prepare effective enabling prompts, but also it seems that they are useful in re-engaging students and assisting students who have not been progressing in their learning of mathematics.

One key issue we have noted is the timing of prompts, in that it is important that prompts not be given too early, as students need seek to understand and explore possibilities. Likewise, the task should not be reduced more than one conceptual step at a time because the aims are to keep students engaged at their optimal level and to enable them to tackle the original tasks and hence remain a part of the learning community.

As an aside, it is noted that one of the tasks of the teacher when engaged in such one-on-one interactions is to monitor the work of the students. Especially if the students are working on open-ended tasks, it is possible that they might be reinforcing misconceptions. Associated with the formulation of enabling prompts, is the need to attend to the work the student has completed.

Conclusion

Enabling prompts enhance those treasured opportunities to interact with students one-on-one. Of course, the first step is a commitment to create opportunities for such interactions, but once the teachers are ready to assess individual needs before they interact, they have the opportunity to pose enabling prompts. Assessment, here, may depend on prior knowledge of likely difficulties (facilitating pre-lesson planning) or observation and analysis while students work (facilitating on-the-spot task adaptation).

The title of this paper signalled that we are developing some guidelines for teachers when evaluating factors that contribute to the complexity of a task. We propose that it is possible that task complexity might be a result of: the number of steps involved; the number of variables involved; the modes of communicating responses; the number of elements in recording; the degree of abstraction or visualisation required; the size of the numbers to be manipulated; the language being used; or psychomotor considerations. Thus a teacher could anticipate difficulties, and prepare prompts and associated resources as appropriate to:

- reduce the required number of steps;
- reduce the required number of variables;
- simplify the modes of representing results;
- reduce the written elements in recording;
- make the task more concrete;

- reduce the size of the numbers involved;
- simplify the language; or
- reduce the physical demand of any manipulatives.

Any one of these strategies could be used to engage students in working on related tasks, with the intention that they will subsequently proceed with the main task.

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