Students' Willingness to Engage with Mathematical Challenges: Implications for Classroom Pedagogies

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As part of a project exploring various aspects of teachers' choice and use of challenging mathematics tasks, we sought some responses from students on their preferences for the difficulty of tasks on which they might work and also on the ways of working. Despite the common finding that teachers are reluctant to pose challenges to their students for fear of adverse reactions, many students reported that they prefer tasks to be somewhat challenging and many prefer to work on the tasks before having the process explained by the teacher. An important finding was the diversity of student preferences. There are implications for the information that educators offer to teachers on structuring their lessons.

A fundamental assumption of the research reported here is that the learning of mathematics involves students exploring networks of interconnected ideas by engaging with experiences that are appropriately challenging and which require some degree of risk taking by the students (and perhaps by the teacher). There is evidence (e.g., Sullivan, Doug Clarke, & Barbara Clarke, 2013) though that teachers are often reluctant to pose challenging tasks to their classes for fear of negative student reactions. We are exploring the extent to which such fears are well founded.

We apply the term *challenging* to tasks that require students to process multiple pieces of mathematical information simultaneously and make connections between them, and for which there is more than one possible solution or solution method. Our expectation is that students have not been shown a procedure to follow before engaging with the tasks, otherwise the nature of the challenge and the capacity of students to build the connections for themselves is reduced. At least part of the challenge is the expectation that students record the steps in their solutions, explain their strategies and justify their thinking to the teacher and other students. Such challenges require students to *persist.* In this context, we mean that, when confronted by a task that requires them to make decisions on the goal and solution strategy, the students do not appeal to the teacher for direction but seek to solve the task for themselves, even if the solution pathway is not obvious.

The following draws on an aspect of a larger project[i] in which we are examining what happens when teachers encourage their students to persist and ways that students respond to the challenges. In this paper, we present responses to survey items that indicate that, rather than being reluctant to take up mathematical challenges, students prefer them. There is, though, a diversity of student preferences that must be considered by teachers when planning and enacting their lessons.

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Framework Informing the Research

The data reported below are informed by a framework, adapted from Clark and Peterson (1986), which proposes that the teachers' intentions to act are informed by their knowledge, their disposition, and the constraints they anticipate experiencing. Teachers then enact those intentions in their classrooms. We note that even though this framework describes influences on teacher intentions and the data are from students, it is the teachers' anticipation of student preferences that is at issue and we are hoping to explore the validity of the apparent underlying assumptions of the teachers. As Sullivan, David Clarke, and Doug Clarke (2012) argued, teachers' planning decisions are strongly influenced by their judgments about their students. It therefore makes sense to interpret data from students in the context of a framework that describes influences on teachers' decisions. The following explains each of the elements of the framework in the context of task use and how teacher actions and student responses are connected.

Teacher knowledge – One aspect of teachers' knowledge that influences the choice and use of challenging tasks is whether teachers know the relevant mathematics sufficiently well to allow them to be flexible (Hill, Ball, & Schilling. 2008). A second aspect relates to pedagogical approaches that can facilitate learning based on students' working on challenging tasks (Hill et al., 2008). Even though no data are presented here on the mathematical or pedagogical knowledge of project teachers, it is noted that we worked through each task with the project teachers including being explicit about the mathematical purpose of the suggested tasks. We were also explicit about the particular pedagogies we recommended.

Teacher disposition to task use – Influences on teachers' disposition toward the choice and use of tasks include the beliefs that teachers hold about the nature of mathematics, processes by which students come to learn mathematics (e.g., Thompson, 1992), and teachers' attitudes to mathematics and its learning (e.g., Hannula, 2004). A further critical disposition is the willingness to allow students to struggle. Various studies have noted the tendency for teachers to reduce the demand of tasks when planning (Tzur, 2008), and to over explain tasks during lessons (e.g., Stein, Grover, & Henningsen, 1996), in both cases fearing negative student reactions. Similarly, Smith and Stein (2011) described a hierarchy of classroom tasks moving from "Memorization" to "Procedures without connections" to "Procedures with connections" to "Doing Mathematics" tasks. Smith and Stein argued that teachers tended, for example, to adapt tasks with potential for Doing Mathematics down to Procedures with Connections based on anticipated student reactions, including their willingness to take up the challenges. It is such decisions on adapting tasks that we are exploring.

Constraints teachers might anticipate – Connected to teachers' dispositions, and central to the data below, are the constraints teachers anticipate they will experience in implementing challenging tasks in their classrooms. Some of these constraints include the diversity of students' cultural backgrounds (Delpit, 1988), and the skill and language levels of some of the students might inhibit their willingness to engage with challenging tasks (see Stein & Lane, 1996).

A major constraint seems to be the anticipation by teachers of students' unwillingness to take risks and to persist. This was identified by Desforges and Cockburn (1987) who argued that students can resist challenging tasks by threatening classroom order. Dweck

(2000) explained that those students who are more likely to avoid persisting have a performance orientation, meaning they seek social affirmation rather than understanding of the content. Dweck claimed that teachers can inadvertently encourage such responses by affirming easy successes and by failing to affirm effort. Dweck also suggested that teachers can foster a willingness to persist in students. While clearly teachers need to support students to overcome the challenges, and the project offers suggestions of how teachers might do this, the teachers' anticipation of student reactions is an important variable in understanding the ways that tasks are implemented by teachers.

Intentions and action – Based on these sets of influences, teachers formulate intentions to act. It is helpful for teachers to adopt lesson structures that support the use of challenging tasks effectively, and to ensure that students appreciate the intention of the tasks and the relationship of challenge and persistence to their learning. While there are many key pedagogical actions, perhaps the most critical is for the teacher to observe the individuals and groups, offering assistance for those who need it, posing challenges for those who are ready, and selecting students whose solutions can productively contribute to the whole class discussion phase of lessons. Similar advice was offered by Smith and Stein (2011) who described five pedagogical actions that follow the choosing of the task, specifically: anticipating potential student responses; monitoring responses interactively; selecting representative responses for later presentation; sequencing those responses; and connecting the students' strategies with the mathematical processes that were the intention of the selection of the task in the first place. Teachers' intentions to take such actions are no doubt informed by the responses they anticipate from the students.

The Context and Process of Data Collection

In gathering the data reported below, we worked with teachers in years 5 to 8 (ages 10 to 14) in schools serving communities from a variety of socio economic backgrounds.

We adopted a design research approach which "attempts to support arguments constructed around the results of active innovation and intervention in classrooms" (Kelly, 2003, p. 3). The key elements are that we are *intervening* to prompt (possibly) different pedagogies from those used normally, our approach is *iterative* in that subsequent interventions are based on previous ones, and the intent is that findings address issues of *practice*, in this case awareness of student preferences about the level of difficulty of the mathematics tasks on which they might work.

For the first iteration, at the start of the school year we met with the teachers for two days to explore issues associated with student motivation and persistence and to work through a set of tasks. We pre-tested the students and invited them to complete some survey questions. Similar responses were sought on a post-test. We then repeated the entire process for iteration 2, and some results from this iteration are presented below.

In the test and survey, the students were invited to respond to prompts that were similar to tasks that the teachers had worked through and which the students had also experienced as classroom lessons by the time they responded to the post-test. The relevant lesson suggested for years 7 and 8 was based on the following task, referred to below as "tickets":

> The cost of tickets for 2 adult and 5 children to the football is \$65. One adult and 2 children's tickets cost \$29. How much does an adult ticket cost? Represent your solution in two DIFFERENT methods. One of your methods should use equations.

Were this posed as part of a focus on simultaneous equations (which commonly is taught in Year 9), it would not be challenging. However, the task was posed as part of a set of lessons on linear equations and the students had not been taught a procedure for answering such questions. A further aspect of the challenge was that the students were asked to create two different solution strategies for themselves such as those that used diagrams and those that used more formal representations such as equations.

The relevant lesson for years 5/6 students was based on this task, described as "taxi":

> The taxi fare has a \$3 flagfall (what you pay when you get into the taxi) and then \$1.50 per km after that. It does not matter how many people are in the taxi. Mike and Matt do not know each other but decide to share a taxi because they are going in the same direction. The journey to Mike's house is 20 km, then a further 30 km to Matt's house. How much should each of them pay for the taxi? Explain why your suggestion is fair for both people.

In this task, in addition to processing considerable information, there is ambiguity in the solution and different possible solutions are at least arguable. For this reason we describe this task as challenging. It is noted that we were explicit with the teachers about the pedagogical actions that we proposed. These are elaborated further below.

The overall question being explored is "Are teachers' fears of negative student reactions well founded?" The sub-questions informing this aspect of the project, and which formed the focus of the data collection, were:

- Did the students learn the anticipated mathematics from engagement with the proposed lessons?
- What are the students' preferences for the level of difficulty of classroom tasks?
- What are students' preferred ways of working on challenging mathematics tasks?

Results

Both before the teaching started and after the teaching of the lessons, the students completed an online test that included some survey questions. The items were developed using similar phrasing and complexity of distracters as those in the published NAPLAN (the Australian numeracy test) items.

The secondary students were invited to indicate which of four options (7 g, 50 g, 30 g, 20 g) was the answer to this question, termed "eggs":

> A container and 3 eggs weigh 170 grams. The same container and 5 eggs weigh 270 grams. What is the weight of the container?

The year 5 and 6 students were asked to answer the following question, termed "MYKI", given the options \$15, \$6.50, \$19, \$10.

> You need a MYKI card before you can travel on public transport in Melbourne. It costs \$4 to buy a MYKI card and you need to put extra cash on the card to travel. If each journey costs \$2.50, what is the total cost of 6 journeys?

The intention was that the responses to these items would indicate whether the students learned the anticipated strategies from working on the "tickets" and "taxi" tasks respectively. In retrospect the MYKI item drew on a context that was perhaps overly specific, but the MYKI card and its use had been widely publicised in Victoria at the time of the testing and the item was the same on both tests. Nevertheless the results should be read in awareness of some possible ambiguity.

Table 1 presents the comparison of correct responses to each item on both the pre- and post-test, along with the number of responses in each case.

Table 1
Comparison of the Percentage of Students (and the number) who were Correct on the Relevant Pre- and Post-Test Items

	Pre test	Post test
Eggs (Year 7 – 8)	48% (n = 360)	61% (n = 285)
MYKI (years 5 – 6)	32% (n = 892)	46% (n = 777)

While the differences between the means in both cases are statistically significant (t_{eggs} 3.31, df (est) = 284, $p < 0.001$, $t_{MYKI} = 5.52$, df (est) = 776, $p < 0.001$) due to the large numbers in the samples, it is important to interpret what the gains might mean for student learning. On one hand, it might be anticipated that working on the specific lessons that we suggested would produce greater improvement. On the other hand, improvement growth is slow overall as an inspection of changes over time in facility of NAPLAN items indicates. It is also possible that the online multiple choice format is not a reliable measure of student learning. Nevertheless the improvement, such as it is, is indication of student learning. On both surveys, and directly following their responses to the above items, students were also asked to choose

a) one of three options to indicate the degree of task difficulty that they preferred; and
b) one of three options to indicate their preferred ways of working.

While the responses to these items are of interest separately, in the interests of saving space tables 2 (for secondary students) and 3 (for the primary students) present the cross tabulation of the number of student responses from the post-test survey.\

In reading the table, the bottom row labelled “total” indicates that 63% (184/293) of these secondary students across a range of schools and teachers, prefer classroom tasks to be as “hard” as this one (which we consider to be “Doing mathematics”) and a further 19% prefer them to be harder. Taken on face value this suggests that, far from being reticent to engage with challenging tasks, most of these students welcome the opportunity.

While the trends in the table are highly statistically significant (Chi square = 44.0, 4 degrees of freedom, $p = 0.00$) it is the numbers in the respective cells that are of interest. Note that the table was completed after the students had experienced a lesson based on a similar task and so they would have been aware of its level of difficulty. Note also that the distribution of responses on the pre-test was quite similar, as were the distributions both times on similar data gathered as part of iteration 1, indicating that we can have confidence that the profile of responses fairly represent the spread of student opinions overall.

There are students in each of the nine cells. This indicates that a teacher might find in their class some students who prefer hard questions but also like to listen to explanations first, and some prefer easy questions and like to work on them by themselves, and all the other combinations as well. It is preferable that teachers are aware that they need to be responsive to a diversity of student orientations while being aware of the preferences of the majority. There is a similar distribution of responses from primary students as presented in Table 3.

Table 2
Cross Tabulation of the Number of Responses to Two Survey Items on the Secondary Post Test

	I prefer the questions we work on in class to be much harder than the egg one.	I prefer the questions we work on in class to be about as hard as the egg one.	I prefer the questions we work on in class to be much easier than the egg one.	Total
I prefer learning how to do questions like the egg question through working by myself	37	48	5	90
I prefer learning how to do questions like the egg question through working with other students	15	89	21	125
I prefer learning how to do questions like the egg question by listening to explanations from the teacher before I work on the question	4	47	17	68
Total	56	184	43	293

Table 3
Cross Tabulation of Two Survey Items on the Primary Post Test

	I prefer the questions we work on in class to be much harder than the MYKI question	I prefer the questions we work on in class to be about as hard as the MYKI question	I prefer the questions we work on in class to be much easier than the MYKI question	Total
I prefer learning how to do questions like the MYKI question working by myself	329	123	15	487
I prefer learning how to do questions like the MYKI question working with other students	60	114	24	198
I prefer learning how to do questions like the MKYI question by listening to explanations from the teacher before I work on the question	28	45	20	93
Total	417	282	59	758

In this case, 55% (417/758) of the primary students, across a range of schools and teachers, prefer tasks to be harder than "MYKI" and 37% prefer classroom tasks to be as "hard" as it. Reading the right hand column, we see that 64% of the students indicated a preference for working by themselves and a further 26% for working with other students. Only 12% indicated a preference for listening to the teacher first. In other words, the majority of the students want to work on tasks at least this difficult before having the solution strategy explained by the teacher.

While the trends in this table are similarly highly statistically significant (Chi square = 132.6, 4 degrees of freedom, p = 0.00) with the direction of the trend obvious from the data, again there are some students in each of the cells. In both tables, the overall trends and the diversity of responses are important for informing decisions about pedagogy.

Summary and Conclusion

The above presented responses of students to content items that matched classroom lessons they had experienced and their expressed preferences for task difficulty and ways of working on the tasks. The data were collected as part of a larger project exploring issues associated with the posing of mathematics tasks that are appropriately challenging.

While the improvement in the facility of the items from pre- to post- was significant and indicative of student learning as a result of the experience, the gains were not necessarily compelling. There may have been reasons for this, but processes for identifying improvement in student learning require further investigation.

The majority of both primary and secondary students expressed preferences for classroom tasks at least as difficult as the task on the test that they had just answered. This is an important aspect in that we can assume that students interpreted the level of difficulty as it was intended. Likewise the large majority of both primary and secondary students indicate they would prefer to work on the task prior to explanations from their teachers. There was a larger than expected proportion of students who indicated they preferred to work on the tasks by themselves. This confirms that the focus of the project is appropriate, specifically seeking ways for teachers to support students when they engage with challenging tasks. It also emphasises that teachers do not need to fear negative student reactions if they pose tasks that are appropriately challenging. There is no need for teachers to minimise the challenge of tasks by thinking that the students will not want to do them. The reasons for the common finding that teachers do limit the demand of tasks requires further investigation.

An interesting result was the diversity of combinations of student preferences. This suggests that teachers need to find ways to respond to the individual needs of students.

Overall the results affirm the pedagogical advice that we offer to teachers, which is, in summary:

- discuss with students the importance of persistence when engaging in challenging tasks;
- pose tasks that are appropriately challenging and allow students time to find their own approaches to the task;
- differentiate the student experience through the use of enabling and extending prompts (see Sullivan, Mousley, & Zevenbergen, 2009) for those students who cannot proceed or those who complete the task quickly;

- conduct class reviews (see Smith & Stein, 2011) which draw on students' solutions to promote discussions of similarities and differences; and
- pose consolidating tasks (see Dooley, 2012), which are similar in structure and complexity to the original task, with which all students can engage even if they have not been successful on the original task.

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